

NBPTS

Adolescence and Young Adulthood

Mathematics

STANDARDS

Second Edition



for teachers of students ages 14-18+

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Adolescence and Young Adulthood/Mathematics STANDARDS

(for teachers of students ages 14–18+)

Second Edition

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The world-class schools the United States requires cannot exist without a world-class teaching force; the two go hand in hand. Many accomplished teachers already work in the nation's schools, but their knowledge and skills are often unacknowledged and underutilized. Delineating outstanding practice and recognizing those who achieve it are important first steps in shaping the kind of teaching profession the nation needs. This is the core challenge embraced by the National Board for Professional Teaching Standards™ (NBPTS). Founded in 1987 with a broad base of support from governors, teacher union and school board leaders, school administrators, college and university officials, business executives, foundations, and concerned citizens, NBPTS is a nonprofit, nonpartisan organization governed by a 63-member board of directors, the majority of whom are teachers. Committed to basic reform in education, NBPTS recognizes that teaching is at the heart of education and, further, that the single most important action the nation can take to improve schools is to strengthen teaching.

The National Board's mission is to advance the quality of teaching and learning by:

- maintaining high and rigorous standards for what accomplished teachers should know and be able to do,
- providing a national voluntary system certifying teachers who meet these standards, and
- advocating related education reforms to integrate National Board Certification® in American education and to capitalize on the expertise of National Board Certified Teachers®.

Dedication to this mission is elevating the teaching profession, educating the public about the demands and complexity of accomplished teaching practice, and making teaching a more attractive profession for talented college graduates with many other promising career options.

National Board Certification is more than a system for recognizing and rewarding accomplished teachers. It offers an opportunity to guide the continuing growth and development of the teaching profession. Together with other reforms, National Board Certification is a catalyst for significant change in the teaching profession and in education.

The Philosophical Context

The standards presented here lay the foundation for the Adolescence and Young Adulthood/Mathematics certificate. They represent a professional consensus on the aspects of practice that distinguish accomplished teachers. Cast in terms of actions that teachers take to advance student achievement, these standards also incorporate the essential knowledge, skills, dispositions, and commitments that allow teachers to practice at a high level. Like all NBPTS Standards, this standards document is grounded philosophically in the NBPTS policy statement *What Teachers Should Know and Be Able to Do*. That statement identifies five core propositions.

1) Teachers are committed to students and their learning.

Accomplished teachers are dedicated to making knowledge accessible to all students. They act on the belief that all students can learn. They treat students equitably, recognizing the individual differences that distinguish their students from one another and taking account of these differences in their practice. They adjust their practice, as appropriate, on the basis of observation and knowledge of their students' interests, abilities, skills, knowledge, family circumstances, and peer relationships.

Accomplished teachers understand how students develop and learn. They incorporate the prevailing theories of cognition and intelligence in their practice. They are aware of the influence of context and culture on behavior. They develop students' cognitive capacity and respect for learning. Equally important, they foster students' self-esteem; motivation; character; sense of civic responsibility; and respect for individual, cultural, religious, and racial differences.

2) Teachers know the subjects they teach and how to teach those subjects to students.

Accomplished teachers have a rich understanding of the subject(s) they teach and appreciate how knowledge in their subjects is created, organized, linked to other disciplines, and applied to real-world settings. While faithfully representing the collective wisdom of our culture and upholding the value of disciplinary knowledge, they also develop the critical and analytical capacities of their students.

Accomplished teachers command specialized knowledge of how to convey subject matter to students. They are aware of the preconceptions and background knowledge that students typically bring to each subject and of strategies and instructional resources that can be of assistance. Their instructional repertoire allows them to create multiple paths to learning the subjects they teach, and they are adept at teaching students how to pose and solve challenging problems.

3) Teachers are responsible for managing and monitoring student learning.

Accomplished teachers create, enrich, maintain, and alter instructional settings to capture and sustain the interest of their students. They make the most effective use of time in their instruction. They are adept at engaging students and adults to assist their teaching and at making use of their colleagues' knowledge and expertise to complement their own.

Accomplished teachers command a range of instructional techniques and know when to employ them. They are devoted to high-quality practice and know how to offer each student the opportunity to succeed.

Accomplished teachers know how to engage groups of students to ensure a disciplined learning environment and how to organize instruction so as to meet the schools' goals for students. They are adept at setting norms of social interaction among students and between students and teachers. They understand how to motivate students to learn and how to maintain their interest even in the face of temporary setbacks.

Accomplished teachers can assess the progress of individual students as well as the progress of the class as a whole. They employ multiple methods for assessing student growth and understanding and can clearly explain student performance to students, parents, and administrators.

4) Teachers think systematically about their practice and learn from experience.

Accomplished teachers are models of educated persons, exemplifying the virtues they seek to inspire in students—curiosity, tolerance, honesty, fairness, respect for diversity, and appreciation of cultural differences. They demonstrate capacities that are prerequisites for intellectual growth—the ability to reason, take multiple perspectives, be creative and take risks, and experiment and solve problems.

Accomplished teachers draw on their knowledge of human development, subject matter, and instruction, and their understanding of their students, to make principled judgments about sound practice. Their decisions are grounded not only in the literature of their fields but also in their experience. They engage in lifelong learning, which they seek to encourage in their students.

Striving to strengthen their teaching, accomplished teachers examine their practice critically; expand their repertoire; deepen their knowledge; sharpen their judgment; and adapt their teaching to new findings, ideas, and theories.

5) Teachers are members of learning communities.

Accomplished teachers contribute to the effectiveness of the school by working collaboratively with other professionals on instructional policy, curriculum development, and staff development. They can evaluate school progress and the allocation of school resources in light of their understanding of state and local educational objectives. They are knowledgeable about specialized school and community resources that can be engaged for their students' benefit and are skilled at employing such resources as needed.

Accomplished teachers find ways to work collaboratively and creatively with parents, engaging them productively in the work of the school.

The Certification Framework

Using the Five Core Propositions as a springboard, NBPTS sets standards and offers National Board Certification in nearly 30 fields. These fields are defined by the developmental level of the students and the subject or subjects being taught. The first descriptor represents the four overlapping student developmental levels:

- Early Childhood, ages 3–8;
- Middle Childhood, ages 7–12;
- Early Adolescence, ages 11–15;
- Adolescence and Young Adulthood, ages 14–18+.

The second descriptor indicates the substantive focus of a teacher's practice. Teachers may select either a subject-specific or a generalist certificate at a particular developmental level. Subject-specific certificates are designed for teachers who emphasize a single subject area in their teaching (e.g., Early Adolescence/English Language Arts, Adolescence and Young Adulthood/Mathematics); generalist certificates are designed for teachers who develop student skills and knowledge across the curriculum (e.g., Early

Childhood/Generalist, Middle Childhood/Generalist). For some subject-specific certificates, developmental levels are joined together to recognize the commonalities in teaching students at those developmental levels (e.g., Early and Middle Childhood/Art).

Standards and Assessment Development

Following a nationwide search for outstanding educators, a standards committee is appointed for each field. The committees are generally made up of 15 members who are broadly representative of accomplished professionals in their fields. A majority of committee members are teachers regularly engaged in teaching students in the field in question; other members are typically professors, experts in child development, teacher educators, and other professionals in the relevant discipline. The standards committees develop the specific standards for each field, which are then disseminated widely for public critique and comment and subsequently revised as necessary before their adoption by the NBPTS Board of Directors. Periodically, standards are updated so that they remain dynamic documents, responsive to changes in the field.

Determining whether or not candidates meet the standards requires performance-based assessment methods that are fair, valid, and reliable and that ask teachers to demonstrate principled, professional judgments in a variety of situations. A testing contractor specializing in assessment development works with standards committee members, teacher assessment development teams, and members of the NBPTS staff to develop assessment exercises and pilot test them with teachers active in each certificate field. The assessment process involves two primary activities: (1) the compilation of a portfolio of teaching practice over a period of time and (2) the demonstration of content knowledge through assessment center exercises. Teachers prepare their portfolios by videotaping their teaching, gathering student learning products and other teaching artifacts, and providing detailed analyses of their practice. At the assessment center, teachers write answers to questions that relate primarily to content knowledge specific to their fields.

The portfolio is designed to capture teaching in real-time, real-life settings, thus allowing trained assessors from the field in question to examine how teachers translate knowledge and theory into practice. It also yields the most valued evidence NBPTS collects—videos of practice and samples of student work. The videos and student work are accompanied by commentaries on the goals and purposes of instruction, the effectiveness of the practice, teachers' reflections on what occurred, and their rationales for the professional judgments they made. In addition, the portfolio allows candidates to document their accomplishments in contributing to the advancement of the profession and the improvement of schooling—whether at the local, state, or national level—and to document their ability to work constructively with their students' families.

Teachers report that the portfolio is a professional development vehicle of considerable power, in part because it challenges the historic isolation of teachers from their peers. It accomplishes this by actively encouraging candidates to seek the advice and counsel of their professional colleagues—whether across the hall or across the country—as they build their portfolios. It also requires teachers to examine the underlying assump-

tions of their practice and the results of their efforts in critical but healthy ways. This emphasis on reflection is highly valued by teachers who go through the process of National Board Certification.

The assessment center exercises are designed to complement the portfolio. They validate that the knowledge and skills exhibited in the portfolio are, in fact, accurate reflections of what candidates know and can do, and they give candidates an opportunity to demonstrate knowledge and skills not sampled in the portfolio because of the candidate's specific teaching assignment. For example, high school science teachers assigned to teach only physics in a given year might have difficulty demonstrating in their portfolio a broad knowledge of biology. Given that the NBPTS Standards for science teachers place a high value on such capabilities, another strategy for data collection is necessary. The assessment center exercises fill this gap and otherwise augment the portfolio. Each candidate's work is examined by trained assessors who teach in the certificate field.

The National Board for Professional Teaching Standards believes that a valid assessment of accomplished practice must allow for the variety of forms sound practice takes. It must also sample the range of content knowledge that teachers possess and must provide appropriate contexts for assessments of teaching knowledge and skill. Teaching is not just about knowing things; it is about the use of knowledge—knowledge of learners and of learning, of schools and of subjects—in the service of helping students grow and develop. Consequently, NBPTS believes that the most valid teacher assessment processes engage candidates in the activities of teaching—activities that require the display and use of teaching knowledge and skill and that allow teachers the opportunity to explain and justify their actions.

In its assessment development work, NBPTS uses technology for assessment when appropriate; ensures broad representation of the diversity that exists within the profession; engages pertinent disciplinary and specialty associations at key points in the process; collaborates closely with appropriate state agencies, academic institutions, and independent research and education organizations; establishes procedures to detect and eliminate instances of external and internal bias with respect to age, gender, and racial and ethnic background of teacher-candidates; and selects the method exhibiting the least adverse impact when given a choice among equally valid assessments.

Once an assessment has been thoroughly tested and found to meet NBPTS requirements for validity, reliability, and fairness, eligible teachers may apply for National Board Certification. To be eligible, a teacher must hold a baccalaureate degree from an accredited institution; have a minimum of three years' teaching experience at the early childhood, elementary school, middle school, or high school level; and have held a valid state teaching license for those three years or, where a license is not required, have taught in schools recognized and approved to operate by the state.

Strengthening Teaching and Improving Learning

The National Board's system of standards and certification is commanding the respect of the profession and the public, thereby making a difference in how communities and policymakers view teachers, how teachers view themselves, and how teachers improve their practice throughout their careers. National Board Certification has yielded such

results in part because it has forged a national consensus on the characteristics of accomplished teaching practice in each field. The traditional conversation about teacher competence has focused on beginning teachers. The National Board for Professional Teaching Standards has helped broaden this conversation to span the entire career of teachers.

Developing standards of accomplished practice helps to elevate the teaching profession as the standards make public the knowledge, skills, and dispositions of accomplished teachers. However, making such standards the basis for National Board Certification promises much more. Since National Board Certification identifies accomplished teachers in a fair and trustworthy manner, it can offer career paths for teachers that will make use of their knowledge, wisdom, and expertise; give accomplished practitioners the opportunity to achieve greater status, authority, and compensation; and accelerate efforts to build more successful school organizations and structures.

By holding accomplished teachers to high and rigorous standards, National Board Certification encourages change along several key fronts:

- changing what it means to have a career in teaching by recognizing and rewarding accomplished teachers and by making it possible for teachers to advance in responsibility, status, and compensation without having to leave the classroom;
- changing the culture of teaching by accelerating growth in the knowledge base of teaching, by placing real value on professional judgment and accomplished practice in all its various manifestations, and by encouraging teachers to search for new knowledge and better practice through a steady regimen of collaboration and reflection with peers and others;
- changing the way schools are organized and managed by creating a vehicle that facilitates the establishment of unique teacher positions, providing accomplished teachers with greater authority and autonomy in making instructional decisions and greater responsibility for sharing their expertise to strengthen the practice of others;
- changing the nature of teacher preparation and ongoing professional development by laying a standards-based foundation for a fully articulated career development path that begins with prospective teachers and leads to accomplished teachers;
- changing the way school districts think about hiring and compensating teachers by encouraging administrators and school boards to reward excellence in teaching by seeking to hire accomplished teachers.

Although National Board Certification has been designed with the entire country in mind, each state and locality decides for itself how best to encourage teachers to achieve National Board Certification and how best to take advantage of the expertise of the National Board Certified Teachers in their midst. Across the country, legislation has been enacted that supports National Board Certification, including allocations of funds to pay

for the certification fee for teachers, release time for candidates to work on their portfolios and prepare for the assessment center exercises, and salary supplements for teachers who achieve National Board Certification. Incentives for National Board Certification exist at the state or local level in all 50 states and in the District of Columbia.

As this support at the state and local levels suggests, National Board Certification is recognized throughout the nation as a rich professional development experience. Because National Board Certification provides states and localities with a way to structure teachers' roles and responsibilities more effectively and to allow schools to benefit from the wisdom of their strongest teachers, National Board Certification is a strong component of education reform in the United States.

Today's students will live and work in tomorrow's world. Teaching them to use mathematics in new and meaningful ways is an exciting and challenging task. Today's teachers are moving away from tedious drills on narrow skills; away from the notion that mathematics comes packaged in neat, self-sustaining compartments; and away from the attitude that only the privileged can understand and do mathematics. Students across the educational spectrum today find their analytic ability to analyze and frame situations they encounter every day in a mathematical context to be at a premium. These students need an understanding of fundamental mathematical concepts and techniques in order to achieve a full command of state-of-the-art technologies to help them solve problems that come up in daily life.

Against this backdrop, the teacher of mathematics at the adolescence and young adulthood level plays a key role—a role that is evolving as more is learned about students and mathematics. The mathematics teacher leads students in an exploration of the world of mathematics, provides learning opportunities for all students, and serves as an advocate for each student in the mathematics classroom. The mathematics teacher opens doors to a wide range of learning possibilities and future options.

In recent years, mathematics has undergone many changes both as a discipline and as a tool for educational, social, and economic opportunity. The development of modern technology has profoundly affected the ways organizations and individuals around the world achieve their objectives. Such technological advances have substantially broadened the applications of mathematics, not only in scholarship, science, industry, business, and government, but also more prominently than before in such areas as agriculture, health, politics, and the arts.

The changing nature of the workforce in the United States, coupled with stiff international competition, requires that all citizens—not just a select few—become mathematically literate. All young adult Americans must master mathematical ways of thinking if they are to succeed and if they are to make informed decisions about issues central to their lives and to society. The mathematical principles and concepts that today's young adults need to know to participate fully in the modern world are both different from and more complex than those required of earlier generations.

These transforming events and new realities are highlighted in several national reports that provide recommendations for the reform of school mathematics and its teaching.¹ Each report asserts that high priority must be given to the development of students who will mature into adults with strong mathematical knowledge and with skills and confidence in their abilities to do significant mathematics.

The recent and continuing changes in mathematics, and in the field's current vision of mathematics instruction, demand the most from the nation's teachers. This conception of mathematics teaching is predicated on the belief that all students—regardless of background, gender, race, ethnicity, special needs, or language—must be afforded the opportunity to achieve the level of mathematical competence required to succeed in life and in work. Giving students access to the full range of school mathematics, such as that outlined by the National Council of Teachers of Mathematics (NCTM) in *Principles and Standards for School Mathematics*,² will require a great deal more, especially more knowledge, from tomorrow's teachers than is typically required from teachers today.

It is important to emphasize that these standards recognize, reflect, and imply varied approaches to teaching. The National Board understands that such factors as the contexts in which teachers practice, the backgrounds and experiences they bring to their role, and

1. The National Commission on Mathematics and Science Teaching for the 21st Century, *Before It's Too Late* (Washington, D.C.: Author, 2000).
Mathematical Association of America, *A Call for Change* (Washington, D.C.: Author, 1991).
Mathematical Sciences Education Board, *Everybody Counts* (Washington, D.C.: National Academy Press, 1989).

2. National Council of Teachers of Mathematics, *Principles and Standards for School Mathematics* (Reston, Va.: Author, 2000).

the professional choices they make during their careers contribute to a diverse population of accomplished secondary mathematics teachers. For example, a teacher who has spent the past 10 years teaching algebra, and who has developed special expertise in alternative methods of assessment, offers a distinctly different profile of accomplished teaching from that of a teacher who has taught the full spectrum of secondary mathematics courses, including calculus, and who has a special interest in cooperative learning. Both, however, may well be teaching at the highest levels described in these standards.

At the same time, however, certain guiding principles relevant to mathematics content and pedagogical knowledge are widely agreed on among mathematics educators. These guiding principles represent the common ground that unites accomplished teachers and distinguishes their practice without regard to their current assignment, school context, or past experiences. Included in these principles are not only understandings about the essentials of first-rate practice, but also understandings about how to recognize and avoid damaging and ineffective practices.

At present, many accomplished teachers are meeting these challenges. Those entering the teaching profession continue to join them in working to achieve the vision set by the NCTM standards. The Carnegie Task Force on Teaching as a Profession³ had a similar vision when it advanced the idea of publicly certifying the work of exceptional teachers, teachers who are making a difference in the lives and abilities of the children they teach. The task force's report stimulated the formation of the National Board for Professional Teaching Standards, which ultimately led to the development of the standards in this document—standards intended to recognize teachers of adolescents and young adults who ensure their students' progress toward high achievement in mathematics.

3. Carnegie Task Force on Teaching as a Profession, *A Nation Prepared: Teachers for the 21st Century* (Washington, D.C.: Author, 1986).

Developing High and Rigorous Standards for Accomplished Practice

In 1990, a committee of Adolescence and Young Adulthood/Mathematics teachers and other educators with expertise in this field began the process of developing advanced professional standards for teachers of students ages 14 to 18+. The Adolescence and Young Adulthood/Mathematics Standards Committee was charged with translating the Five Core Propositions of the National Board for Professional Teaching Standards into a standards document that defines outstanding teaching in this field.

In 2000, a committee comprising original committee members and a new group of educators (including National Board Certified Teachers) was convened to examine and update as necessary the published *Adolescence and Young Adulthood/Mathematics Standards*. This second edition of the standards is the result of the committee's deliberations at meetings and their input into working drafts of the standards.

This NBPTS Standards document describes in observable form what accomplished teachers should know and be able to do. The standards are meant to reflect the professional consensus at this point about the essential aspects of accomplished practice. The deliberations of the Adolescence and Young Adulthood/Mathematics Standards Committee were informed by various national and state initiatives on student and teacher standards that have been operating concurrently with the development of NBPTS Standards. As the

understanding of teaching and learning continues to evolve over the next several years, *Adolescence and Young Adulthood/Mathematics Standards* will be updated again.

An essential tension of describing accomplished practice concerns the difference between the analysis and the practice of teaching. The former tends to fragment the profession into any number of discrete duties, such as designing learning activities, providing quality explanation, modeling, managing the classroom, and monitoring student progress. Teaching as it actually occurs, in contrast, is a seamless activity.

Everything an accomplished teacher knows through study, research, and experience is brought to bear daily in the classroom through innumerable decisions that shape learning. Teaching frequently requires balancing the demands of several important educational goals. It depends on accurate observations of particular students and settings. And it is subject to revision on the basis of continuing developments in the classroom. The professional judgments that accomplished teachers make also reflect a certain improvisational artistry.

The paradox, then, is that any attempt to write standards that dissect what accomplished teachers know and are able to do will, to a certain extent, misrepresent the holistic nature of how teaching actually takes place. Nevertheless, the fact remains: Certain identifiable commonalities characterize the accomplished practice of teachers. The 12 standards that follow are designed to capture the craft, artistry, proficiency, and understandings—both deep and broad—that contribute to the complex work that is accomplished teaching.

The Standards Format

Accomplished teaching appears in many different forms, and it should be acknowledged at the outset that these specific standards are not the only way it could have been described. No linearity, atomization, or hierarchy is implied in this vision of accomplished teaching, nor is each standard of equal weight. Rather, the standards are presented as aspects of teaching that are analytically separable for the purposes of this standards document but that are not discrete when they appear in practice.

The document follows a two-part format for each of the 12 standards:

- I. *Standard Statement***—This is a succinct statement of one vital aspect of the practice of the accomplished Adolescence and Young Adulthood/Mathematics teacher. Each standard is expressed in terms of observable teacher actions that have an impact on students.
- II. *Elaboration***—This passage provides a context for the standard, along with an explanation of what teachers need to know, value, and do if they are to fulfill the standard. The elaboration includes descriptions of teacher dispositions toward students, their distinctive roles and responsibilities, and their stances on a range of ethical and intellectual issues that regularly confront them.

Adolescence and Young Adulthood/Mathematics STANDARDS

(for teachers of students ages 14–18+)

Second Edition

OVERVIEW

The National Board for Professional Teaching Standards has developed the following 12 standards of accomplished practice for Adolescence and Young Adulthood/Mathematics teachers. The standards have been ordered as they have to facilitate

understanding, not to assign priorities. They each describe an important facet of accomplished teaching; they often occur concurrently because of the seamless quality of teaching. The standards serve as the basis for the National Board Certification in this field.

Commitment

I. Commitment to Students and Their Learning (p. 7)

Accomplished mathematics teachers acknowledge and value the individuality and worth of each student, believe that all students can learn and use significant mathematics, and demonstrate these beliefs in their practice.

II. Equity, Diversity, and Fairness (p. 11)

Accomplished mathematics teachers have high expectations for all students. They ensure equal access to the mathematics curriculum; model and promote behavior appropriate in a diverse society by showing respect and appreciation for all students; and teach students to treat one another fairly and with dignity.

Knowledge of Mathematics, Students, and Teaching

III. Knowledge of Mathematics (p. 15)

Accomplished mathematics teachers have a broad and deep knowledge of the concepts, principles, techniques, and reasoning methods

of mathematics, and they use this knowledge to set curricular goals and shape their instruction and assessment. They understand significant connections among mathematical ideas and the applications of these ideas to problem solving in mathematics, in other disciplines, and in the world outside of school.

IV. Knowledge of Students (p. 23)

Accomplished mathematics teachers know and care about their students. They use their knowledge about adolescents and adolescent development, and their knowledge about how this development affects the learning of mathematics, to guide their curricular and instructional decisions. They understand the impact of home life, cultural background, individual learning differences, student attitudes and aspirations, and community expectations and values on student learning.

V. Knowledge of Teaching Practice (p. 27)

Accomplished mathematics teachers have an extensive base of pedagogical knowledge and use it to make curriculum decisions, design instructional strategies and assessment plans, and choose materials and resources for mathematics instruction.

The Teaching of Mathematics

VI. The Art of Teaching (p. 31)

Accomplished mathematics teachers stimulate and facilitate student learning by using a wide range of formats and procedures and by assuming a variety of roles to guide students' learning of mathematics.

VII. Learning Environment (p. 35)

Accomplished mathematics teachers help students learn mathematics by creating environments in which students are active learners, show willingness to take intellectual risks, develop confidence and self-esteem, and value mathematics. This environment fosters students' learning of mathematics.

VIII. Ways of Thinking Mathematically (p. 37)

Accomplished mathematics teachers develop students' abilities to reason and think mathematically—to investigate and explore patterns, to discover structures and establish relationships, to formulate and solve problems, to justify and communicate their conclusions, and to question and extend those conclusions.

IX. Assessment (p. 41)

Accomplished mathematics teachers employ a range of formal and informal assessment methods that are ongoing and embedded to evaluate student learning in light of well-defined goals. They employ multiple methods of assessment—including teacher-designed and external assessments, where appropriate—to diagnose learning; plan instruction; and provide opportunities for students to reflect on their strengths and weaknesses in order to revise, support, and extend their individual performance.

Professional Development and Outreach

X. Reflection and Growth (p. 43)

Accomplished mathematics teachers regularly reflect on what they teach and how they teach. They keep abreast of changes and learn new content in mathematics and in mathematical pedagogy, continually seeking to improve their knowledge and practice.

XI. Families and Communities (p. 47)

Accomplished mathematics teachers support and promote the involvement of families in their children's education. They help varied communities understand the role of mathematics and mathematics instruction in today's world, and—to the extent possible—they involve these communities in the support of instruction.

XII. Contributing to the Professional Community (p. 51)

Accomplished mathematics teachers collaborate with peers and other education professionals to strengthen their school's programs, advance knowledge, and contribute to improving practice within the field.

The pages that follow provide elaborations of each standard that discuss the knowledge, skills, dispositions, and habits of mind that describe accomplished teaching in the field.

Commitment

Accomplished mathematics teachers care deeply about their students and are committed to their learning. Although knowledge (Standards III–V), teaching practice (Standards VI–IX), and professional development and outreach (Standards X–XII) are all essential components of accomplished teaching, they are not sufficient without a total commitment that every student be given every opportunity to learn quality mathematics. This commitment is apparent in all aspects of teaching: in setting clear and ambitious goals, selecting deliberate actions designed to achieve them, and including all students in these plans.

Standard I: Commitment to Students and Their Learning

Accomplished mathematics teachers acknowledge and value the individuality and worth of each student, believe that all students can learn and use significant mathematics, and demonstrate these beliefs in their practice.

Accomplished mathematics teachers focus on students, their activities, and their mathematical understanding. They make the phrase “mathematics for all” come alive in their classrooms. They strive to inspire students to work diligently to learn mathematics, and they encourage students to consider making time for learning mathematics a priority. Above all, they do everything possible to ensure that their students learn meaningful and important mathematics. They are genuinely committed to their students, and they let their students know that they find doing and teaching mathematics lively and enjoyable experiences. They take the extra steps required to ensure that students learn, and they encourage students to advance in mathematics as far as possible.

These teachers⁴ teach high-quality mathematics to all students. They know that mathematical understanding⁵ is essential for everyone, and they work to encourage all students to take courses that center on significant mathematics. They respect all their students and help them meet new challenges. Recognizing that each student can gain

increased mathematical understanding, they expect their students to prosper in the study of mathematics, and they communicate that expectation to them.

Teachers help students acquire confidence in learning, doing, and understanding mathematics. They help students become aware of their own learning and show them that there are multiple ways to learn. Accomplished teachers seek to keep all their students involved in learning mathematics. They respect the contributions of each student to the classroom, and they make sure that each student respects the contributions of others. They encourage students to assume responsibility for their own learning. They recognize that the long-range goal of a teacher is to help students become self-directed and capable of learning on their own.

Teachers care about students as individuals, and they demonstrate this concern through their words and actions. They are alert and sensitive to the variations that exist in students’ prior learning experiences, individual learning approaches, family and cultural backgrounds, interests, and special

4. All references to teachers in this document, whether explicitly stated or not, refer to accomplished mathematics teachers of adolescents and young adults.

5. Mathematical understanding, as used in this document, is defined as knowing the interconnections among concepts and facts; knowing the logical reasoning underlying each fact and making logical deductions therefrom; recognizing the coherence of mathematics by being able to place concepts and facts properly in a hierarchical mathematical structure; being fluent in computational skills; and knowing how and when to apply ideas and skills in specific contexts both within and outside mathematics.

Standard II: Equity, Diversity, and Fairness

Accomplished mathematics teachers have high expectations for all students. They ensure equal access to the mathematics curriculum; model and promote behavior appropriate in a diverse society by showing respect and appreciation for all students; and teach students to treat one another fairly and with dignity.

Accomplished mathematics teachers are dedicated to meeting the needs of an increasingly diverse student population. They confront issues of diversity proactively to promote academic and social equity. They actively and positively challenge sexist, racist, and other biased behaviors and stereotypical perspectives, including those directed toward various ethnic groups, regardless of the source. They are keenly aware of the historical perspectives and biases that have created social and academic barriers for students, and they work to remove these obstacles. They maintain high expectations for all learners regardless of gender, race, socioeconomic class, or previous experience. They ensure that their students receive equal opportunities to learn and advance in mathematics, and they act to dispel the notion that not all students are capable of learning mathematics. They consistently communicate their respect for all students and their belief that all students can learn. By example and guidance, they help students learn to treat one another as valued members of the learning community.

Equity

Accomplished teachers create learning environments in which high expectations exist for all students. They know that some students have historically been discouraged from pursuing advanced study in mathematics or have been denied the opportunity to advance

in mathematics because of the mistaken belief that not all students are capable of learning mathematics. Teachers are particularly careful to make sure that women, students with disabilities, students for whom English is a new language, students of low socioeconomic status, and students of color understand that they—like all other students—can succeed in mathematics.

Teachers are aware of the supportive attention that must be given to students who are learning English as a new language. Teachers make sure that such students are able to understand instruction and participate in class and small-group discussions; teachers may also give students who are learning English as a new language alternative assessments and assignments so that their ability to demonstrate understanding and proficiency in mathematics does not depend on their proficiency in English. Likewise, teachers are aware that special modifications to instructional strategies and resources may be necessary for students with exceptional needs. Teachers work to ensure that such accommodations are made so that these students have equitable access to appropriate learning opportunities.

Teachers consider the full range of academic experience and learning styles present among their students and make sure that all students are given the opportunity to reach their full potential in mathematics. They use enrichment activities, creative grouping strategies, peer mentoring, and a range of instructional strategies and materials to ensure that every student has the opportunity to participate meaningfully in mathematics learning and

achieve high goals. (See Standard V—*Knowledge of Teaching Practice*.)

Teachers communicate their confidence in the capabilities of all students through their words and actions—in their interactions with students in class and in written evaluations, in their attention to students’ interactions with one another, in their assignment of students to instructional groups, and in their efforts to forge productive relationships with colleagues and students’ families in order to support students’ mathematical achievement. (See Standard XI—*Families and Communities*.) Teachers constructively address issues affecting instruction, class management, and student participation, ensuring that all students are able to participate meaningfully in mathematics learning.

Diversity

Accomplished teachers value the importance of their students’ diverse cultures and backgrounds. They build on the richness of the heritage and culture of their students and give students opportunities to think in ways that are both culturally familiar and unfamiliar. They recognize the unique contributions and perspectives each student brings to the learning environment. Teachers use this knowledge to foster positive interaction in the classroom and to support each student’s mathematical growth. They are aware that students’ cultural backgrounds and life experiences can influence the way they interact in the classroom and the way students approach and learn mathematics.

Accomplished mathematics teachers show respect for diversity of all kinds. They model the use of unbiased language and set positive examples of acceptance and appreciation of others’ uniqueness. When the need arises, they respond quickly and decisively to negate the effects of biased speech and behavior. They consistently communicate their belief that each individual makes a unique contribution to the character of the learning environment and to society.

Fairness

Accomplished mathematics teachers know that the attitudes they manifest set powerful examples for young people, and they conscientiously demonstrate appropriate behaviors and attitudes that they wish to inculcate in their students. They are conscious of creating and maintaining a community of learners in which all students are treated fairly by their peers and teachers.

Teachers have and demonstrate high expectations for all students and offer all students the opportunity to gain and demonstrate proficiency and understanding. They ensure that every student—regardless of gender, background, culture, ethnicity, sexual orientation, or special needs and abilities—is treated fairly in the learning environment. Teachers allocate resources, including instructional time, fairly, knowing that the fairest division of resources is not always arithmetically equal because of the unique needs of learners.

Accomplished teachers are aware of the issues involved in providing instruction to students with exceptionalities. They modify curriculum, instruction, and assessments as necessary. They comply with federal, state, and local laws, regulations, and policies concerning students with unique needs. They work closely with the specialists and support personnel who have valuable insights into these students and willingly team with them to ensure that these students have every opportunity to achieve their educational goals and objectives. They advocate for and make use of assistive technologies, for instance, computers with voice-recognition or speech-synthesis software that can enable students with exceptional needs to communicate their thought processes and mathematical arguments.



Knowledge of Mathematics, Students, and Teaching

Accomplished teachers offer all students the opportunity to learn. Only by having a deep and broad understanding of mathematics can teachers organize and deliver instruction that helps students build their own broad and deep understanding of mathematics. Only by knowing their students well can teachers consistently make instructional decisions that will further students' learning. Further, only by skillfully combining their knowledge of students and mathematics with their knowledge about how to teach mathematics can teachers enable students to learn mathematics successfully. The following three standards form the foundation for the decisions and actions taken by accomplished mathematics teachers. They are the basis for the seven remaining standards.

Standard III: Knowledge of Mathematics

Accomplished mathematics teachers have a broad and deep knowledge of the concepts, principles, techniques, and reasoning methods of mathematics, and they use this knowledge to set curricular goals and shape their instruction and assessment. They understand significant connections among mathematical ideas and the applications of these ideas to problem solving in mathematics, in other disciplines, and in the world outside of school.

Mathematics is a discipline that develops intellectual power to solve problems; make decisions; and describe significant visual, quantitative, and symbolic patterns. It is a fundamental tool in the persistent human effort to make sense of the world—its order, chaos, stability, and change. It has applications, for example, in scientific, technical, economic, and political work; in art and music; and in our personal lives and social interactions.

Although it is one of the oldest disciplines of human knowledge and thought, the field of mathematics continues to grow and evolve. New concepts, principles, and methods become a part of the discipline each year. For example, the emphasis on computational efficiency, the discovery of new algorithms, the advent of linear programming, and the articulation of the number theory underlying cryptography all have been developed during the lifetime of many of today's teachers.

Mathematics is commonly discussed in terms of computational procedures and proofs in the school subjects of algebra, geometry, and calculus. However, mathematics is a far more complex organism. It embraces input from theoretical physics, computer science, economics, and other applied fields. It strives for generality and, even in applications, makes effective use of abstractions as a source of power. The emergence of sophisticated technology also makes possible the approximate solutions of formerly inaccessible problems by virtue of computer simulations. Technology also contributes to explorations that uncover new phenomena. A full understanding of these new advances then stimulates even more abstract theoretical work.

To make classroom decisions that support student learning, teachers must understand both the mathematics and their students, and they must continue to grow in their understanding. (See Standard I—*Commitment to*

Students and Their Learning for a definition of mathematical understanding.) To help students acquire and then build on the ideas, methods, and skills that underlie important mathematics and see relationships among these elements and make significant applications of them, accomplished teachers must have a broad and well-integrated knowledge of both classical and contemporary ideas, as well as the methods and techniques of mathematics. They must appreciate the richly interconnected nature of the discipline.

Accomplished teachers also view the discipline from several perspectives. They know the central concepts, principles, facts, and techniques of important mathematical domains—algebra, geometry, discrete mathematics, data analysis and statistics, and calculus. They know the fundamental processes of mathematical thinking, including representation, modeling, conjecture, inference, interpretation, and analysis. They understand the fundamental role of proof in establishing and explaining the truth of mathematical statements and in providing a standard of logical connection that sets mathematics apart from other disciplines.

They know the productive connections between mathematics and other fields of human endeavor—connections that have given mathematics a remarkable history of intellectual service to problem solving and decision making across time and cultures. They also understand the ways mathematics is contributing to the technological changes in society and the ways technology is changing the face of mathematics.

Increasingly, in today's classrooms, students engage in mathematical investigations—from simple explorations to open-ended problems aimed at forming conjectures to intensive or long-term researchlike activities. In this environment, teachers should anticipate or respond to the questions and ideas students may develop and to the disruptions or unexpected connections they might encounter. They should acknowledge how long a worthwhile journey might take. Good classroom

judgment—knowing when or whether to intervene, deciding which of many possible avenues will be most fruitful to follow, and knowing when to bring closure to an activity and move on—depends not only on teachers' sensitivity to students' learning, but also on profound mathematical knowledge that allows them to recognize and pursue mathematically significant ideas.

An increased emphasis on applications also puts new demands on teachers' mathematical knowledge. Sometimes responding to these demands means entering unfamiliar mathematical territory. But even when the mathematics itself is not new to the teacher, a shift of curricular organization from mathematical topics to contexts or applications increases the need for teachers to distinguish between those mathematical ideas that are new and important for students and those that are relatively routine.

One of the strongest forces in the contemporary growth and evolution of mathematics—and of mathematics teaching—is the power of modern computational technology. Entirely new mathematical fields are emerging. For example, understanding recursive structures and being able to analyze algorithms become more important for students to know, placing new demands on what teachers must know. Also, some problems and topics are becoming more accessible to students, along with new ways to represent and manipulate mathematical information. This gives teachers new choices about both content and pedagogy. To make wise choices about how to use calculators, computers, and appropriate software in the service of students' mathematical learning, teachers need not only the mathematical knowledge that guides their pursuit of mathematical goals, but also knowledge of the potential of the technology—including the emerging software relevant to their discipline—and fluent expertise with its use.

Core Mathematical Knowledge

Accomplished teachers understand the major ideas in the core domains of mathematics. Although their expertise may vary in degree for particular domains, they have a fundamental knowledge base from which to build student mathematical understanding. This knowledge includes algebra, geometry, discrete mathematics, data analysis and statistics, and calculus.

Algebra

Accomplished teachers recognize algebra as a language for describing abstract mathematical structures and for generalizing and extending ideas from arithmetic. They know how to apply these aspects of algebraic content and thinking to classroom teaching in appropriate ways throughout the mathematics curriculum. Substantive mathematical reasoning applies to such algebraic ideas as identities, inverses, closure, linearity, and the distributive law. These ideas and the systems that use them (e.g., groups, rings, and fields) are foundations for understanding the mathematics of everyday classrooms.

Accomplished teachers also recognize algebra's role as a language for modeling problem situations and for reasoning and drawing inferences about functions and relations. They understand the interplay among numerical, symbolic, verbal, and graphical representations of quantitative relationships and the role and means of transforming and simplifying these representations. They are proficient in using concepts and symbolic expressions for working with families of functions, such as polynomial, exponential, rational, logarithmic, trigonometric, and those that depend on parameters or are recursively defined. They connect these functions to important applications through their special properties.

Many ideas that are central to secondary mathematics have their roots in a more abstract algebra that is not always evident in the standard curriculum. To maintain a coherent view of the standard material, teachers need to know its underpinnings. For example, the fact that the decimal expansions of rational numbers either terminate or repeat can be seen from an examination of the division algorithm, and the never-divide-by-zero rule derives from the algebraic equivalence of two statements, $a/b = c$ and $a = bc$.

Geometry

Accomplished teachers understand that geometry provides a repertoire of techniques for studying spatial objects and patterns as well as a setting for the representation of seemingly purely analytic or algebraic ideas. Just as accomplished teachers use algebraic methods to support reasoning about geometric situations, they use visual models and methods (concrete and virtual, in two and three dimensions) or ideas from non-Euclidean geometry to provide insight into patterns expressed in numbers or other symbols. Using their knowledge of the properties of Euclidean spaces, teachers solve problems in a variety of fields, such as art, architecture, and engineering.

They use approaches that involve measurement (of distance, area, volume, and angle measure, for example), ideas of similarity (proportion and trigonometric ratios, for example), vectors, and coordinates. They also use drawing techniques, including paper/pencil and software, to investigate and analyze properties of geometric objects and to study which properties change and which remain invariant as the objects are transformed in various ways. Accomplished teachers should also be familiar with other areas of mathematics opened up by technology, such as self-similarity in fractal geometry.

Accomplished teachers are proficient in constructing mathematical proofs and in explaining proofs to students. Teachers are able to use the axiomatic structure of geometry to construct proofs in varied forms (e.g., paragraph, indirect, two-column).

Discrete Mathematics

Accomplished teachers' understanding extends beyond the mathematics of continuous phenomena. In the context of this document, discrete mathematics includes such topics as algorithms; sequences, formal series, finite differences, binomial theorem; recursion and iteration; proof by induction; graphs and networks; and counting techniques (e.g., combinatorics, arrangements, partitions). It also includes elements of number theory (e.g., modular systems, divisibility and counting of divisors, integer solutions to problems) and fundamental notions in set theory and logic. Accomplished teachers also understand the foundations and concepts of finite probability (e.g., mutually exclusive, independent, and compound events; conditional probability; expected value).

Accomplished teachers also can use representations appropriate to these domains, including matrices, Venn diagrams, and tree diagrams. They are able to use the ideas and methods of discrete mathematics in such contexts as social choice (e.g., voting, apportionment), finance, population dynamics, optimization (e.g., choosing best paths), and computer science as well as in the analysis of problems within mathematics itself.

Data Analysis and Statistics

Accomplished teachers use both quantitative and qualitative approaches when answering questions involving data. To do so, they collect, organize, represent, and reason about data, using a variety of numerical, graphical, and algebraic concepts and procedures, and

they look for ways to describe and model patterns in data. They know how to interpret and draw inferences from data to make decisions in a wide range of applied problem settings as well as how to use simulations to investigate situations.

Teachers understand the connection between simulations and experimental probability. They also see how probability contributes to an understanding of sample spaces, distributions, and the foundations of inferential statistics.

Accomplished teachers understand the mathematical underpinnings of such basic inferential techniques as confidence intervals and hypothesis testing. In addition, they are aware of the advantages, limitations, and appropriateness of each technique. They understand that statistical inference goes beyond describing data and involves using formal methods to support or refute generalizations about populations based on samples, using the methods and language of probability, and using statistical reasoning to make or modify decisions on the basis of data. Teachers understand that a goal of data interpretation is to help students become more informed consumers of information.

Calculus

Accomplished teachers appreciate the historical development and significance of calculus and know that it provides methods for modeling dynamic change in such areas as the physical, biological, and social sciences, as well as in business applications. They are knowledgeable about the theoretical foundations of calculus, including the rigorous development of calculus concepts such as limits, continuity, differentiation, and integration. They understand and can explain how to use limits, derivatives, integrals, and infinite series as tools to measure and analyze rates of change, optimization, and accumulation of continuously varying quantities. They apply the ideas and techniques of

calculus, using numerical methods to analyze data-based problems. They understand how to use technology appropriately to assist in visualizing and solving problems.

Mathematical Thinking Processes

Mathematics is often described by naming important concepts, facts, and operations in its major topic strands. However, characteristic mathematical thinking processes are used to solve problems in all the topical strands. Accomplished teachers of mathematics understand and are able to demonstrate such mathematical processes as these:

- Discovering, describing, and reasoning about patterns represented in visual, numerical, and symbolic form—including such processes as classification, representation, and deductive and inductive reasoning and such concepts as symmetry, similarity, stability, recursion, and continuity
- Using methods for formal verification of mathematical conjectures—including rules of logical inference and proof strategies
- Modeling mathematical relations in problem situations by using symbolic expressions—representing important relationships, operating on symbolic expressions to gain understanding of the situation or to draw inferences about it, and applying mathematical analysis to solve problems and make decisions
- Using heuristics to solve mathematical problems—such as testing extreme cases, conducting an organized search of specific examples, and using visual problem representations
- Using technology to search for patterns and formulate generalizations

- Applying strategies for communicating mathematical information in verbal, numerical, graphic, and symbolic forms and through physical models of mathematical principles

Certain fundamental thinking processes and mathematical structures apply across all topic strands, giving coherence to the subject and powerful support for teaching, learning, and applied problem solving. For example, matrices are invaluable tools for recording and reasoning about complex data sets in algebra (systems of equations), geometry (transformations), graph theory (edge connections), and probability (Markov chains). The concept of limit—central to and made explicit in calculus—appears in the curriculum prior to calculus in such diverse areas as geometry and functions. The linear algebra concepts of vector space and linear transformations provide representations of concepts, problems, and techniques in both algebra and geometry. The algebraic structures of groups, rings, and fields illuminate the common and fundamental properties of operations in number systems, families of functions, and geometric patterns like symmetry. Teachers must be familiar with such core patterns in the discipline if they are to make the power of these structural connections available to students.

Contexts for Mathematics

In one sense, mathematics is among the most abstract of disciplines. For centuries, it has been cultivated for its intrinsic beauty and merit as an intellectual discipline. But its most interesting abstract concepts, structures, and operations have arisen from or found embodiment in patterns of objects and actions in scientific, technical, economic, or other practical situations. Accomplished teachers understand the roots of abstract concepts and techniques in concrete cases,

and they use this understanding to make wise curricular and instructional decisions and to help students make connections across disciplines. They appreciate the historical course through which mathematical ideas have developed and the ways different cultures have influenced and contributed to that development.

An accomplished teacher's knowledge of the context within which mathematics has evolved and is useful includes the following:

- Knowledge of the major threads in the historical development of key mathematical ideas—the conceptual stumbling blocks and insights that provided important breakthroughs—and the contributions of various individuals and cultures to those developments.
- Knowledge of the ways mathematical ideas have been and are today fundamental to practical and scientific progress in fields related to mathematics. This includes applications for the major concepts, principles, and techniques of core content topics in the school curriculum as well as the modeling processes that are fundamental to effective applications of mathematics. Such applications provide a basis for thinking about and using mathematics. Effective use of technology is an essential part of this modeling and application process.
- Knowledge of a set of analytical and representational techniques, the ability to recognize when the techniques are appropriate, and the ability to apply them in real situations.

Applying Mathematical Knowledge in Teaching

Knowledge of mathematics by itself does not guarantee that an individual will become an accomplished teacher of mathematics. However, it provides an essential foundation that supports all other teaching standards and meets the content-based challenges that will occur in classrooms.

Accomplished mathematics teachers' knowledge of mathematics is constantly growing in a way that encourages the integration of new facts, concepts, procedures, technologies, and applications into their teaching repertoire. They employ this solid knowledge base to design instruction that reflects the diverse historical and cultural roots of mathematics. They convey to their students the spirit of mathematics as a human endeavor that has evolved from the contributions, values, and social perspectives of a wide variety of people across thousands of years.

Their love and enthusiasm for mathematics permeates the classroom environment. They convey the power and fascination of the discipline to their students as they engage them in exploring and discovering the intriguing patterns and processes of mathematics and in applying mathematical ideas in realistic settings. They place a high value on doing mathematics, take joy in it, and communicate that joy and excitement to their students.



Standard IV: Knowledge of Students

Accomplished mathematics teachers know and care about their students. They use their knowledge about adolescents and adolescent development, and their knowledge about how this development affects the learning of mathematics, to guide their curricular and instructional decisions. They understand the impact of home life, cultural background, individual learning differences, student attitudes and aspirations, and community expectations and values on student learning.

Accomplished mathematics teachers are aware of the different ways in which students process information. They also recognize the varied mathematics backgrounds their students may have. As they help students internalize the language of mathematics and its processes, they recognize that students learn through varying approaches. For example, some students are adept at dealing with graphic images, and others are better with abstract symbolic forms. Some students learn most efficiently through quiet individual study, others through written reflections, and others through lively verbal exchanges with classmates. Still others may learn through interactive technologies.

Teachers recognize the merits and limitations of each approach and realize that all students will benefit from a multiplicity of approaches that allow them to consider important mathematical ideas and concepts from several perspectives. In designing lessons, teachers consider how students with differing knowledge, interests, and priorities come to learn and use mathematics in different ways. They are sensitive to how these students develop the reasoning processes and attitudes that characterize mathematical thinking and understanding.

Accomplished teachers build on student strengths and address student weaknesses. They go further to notice those students in all groups who have developed exceptionally high abilities or affinities and tailor their

programs to provide challenges and opportunities that support these students. Their lessons succeed—in part—because of their ability to recognize student strengths and to assess, anticipate, and address student difficulties, understandings, and misconceptions. (See Standard IX—*Assessment*.) The practice of such teachers is distinguished by their capacity to integrate the goals of the curriculum with each student’s knowledge base.

Teachers use their professional experience and knowledge of research to create learning experiences that encourage students to explore and build on previous knowledge and understandings. They structure lessons—including the use of manipulatives, technology, and activities—to enable students to recognize the connections among diverse concepts and multiple representations. (See Standard V—*Knowledge of Teaching Practice*.) They appreciate that students’ learning will be influenced by their experiences both in and out of school, and they structure their teaching to capitalize on students’ responsiveness to issues that have relevance to their lives. They are aware that student attitudes and approaches toward learning mathematics differ. They make decisions designed to facilitate positive, meaningful participation of all students in their classes. They strive to motivate students to work hard.

In addition to knowing how students develop and learn, accomplished teachers diagnose and assess student learning. (See Standard IX—*Assessment*.) They build on



their extensive experience to select techniques and approaches that are most likely to provide students the opportunity for success, regardless of specific needs or interests, while maintaining curricular expectations. They are keen observers of their students. They develop a growing base of practical, classroom-based knowledge they can use continually to refine their teaching practice. Teachers are able to identify areas in which remediation is needed on the basis of each student’s unique mathematical background. They are able to work with students to fill learning gaps that may exist and to solidify mathematical understanding.

To gauge their students’ strengths, needs, talents, and interests, teachers insightfully observe and listen to their students in diverse settings, such as a formal classroom setting, an individual conference, or an informal conversation. These insights—along with their ability to identify exceptionalities in their students—enable accomplished teachers to adapt their practice to meet the needs of every student. Accomplished teachers work collaboratively with specialists, as necessary,

and modify their plans and materials to support every student.

Students from different social and cultural backgrounds may have different views on the relevance of given mathematical problems and on ways to apply mathematics to their lives. Accomplished teachers understand how society and prior school experiences shape students’ attitudes and beliefs about mathematics. They acknowledge the potential influence of gender, race, ethnicity, social class, and language on how students—including those whose primary language is not the language of instruction—approach mathematics. Recognizing that students’ personal histories affect how they relate to mathematics, accomplished teachers help students appreciate the universality of the discipline. And, they find techniques that help students enter, relate personally to, and become competent in that human endeavor.



Reflections on Standard IV:

Standard V: Knowledge of Teaching Practice

Accomplished mathematics teachers have an extensive base of pedagogical knowledge and use it to make curriculum decisions, design instructional strategies and assessment plans, and choose materials and resources for mathematics instruction.

Accomplished mathematics teachers design their lessons with important mathematical goals in mind. They articulate these goals clearly, and they select instructional techniques and activities that enable students to meet them. They know their students, and they know the ideas and procedures of mathematics. They also know that mathematical thinking evolves in the minds of individual students based on their experiences. They know the problems and difficulties that students commonly encounter when studying various mathematical topics.

Accomplished teachers apply this knowledge to make judgments about content choice, sequence, emphasis, and instruction. Their repertoire of teaching strategies—including inquiry, cooperative learning, discovery, directed instruction, individualized instruction, and group instruction—engages students in exploring, discovering, and using mathematical ideas. The tasks and activities they select are structured deliberately to facilitate student understanding, communication, and reasoning.

Accomplished teachers understand that not all instructional formats and materials are appropriate for—nor will they appeal to—all students, and they structure their teaching in a variety of ways to address this diversity. Because they understand different types of representational models and the strengths and weaknesses of each, they can select those best suited for different students and for different teaching situations. Thus, they provide students with a variety of tools to solve problems, and they know how to use

those tools appropriately. They help students learn about learning mathematics.

Accomplished teachers have developed a framework of mathematical goals and instructional strategies that allows them to identify, assess, adapt, and create instructional resources to support and enhance student learning. They use a variety of activities and materials to reach their mathematical goals, and they draw on these resources to meet the mathematical needs of their students. These resources include manipulative tools, printed materials, human resources, historical material, appropriate technology, and library and media resources.

The visual, computational, and interactive power of modern technology can be used to influence both what is taught and how students learn mathematics. Accomplished teachers recognize the opportunities afforded by the new tools—access to new ideas and new ways of representing and manipulating them—and effectively use the tools to deepen and enrich students' mathematical learning. They continually improve their own skills and fluency with these tools and reexamine their teaching practice in light of what the tools make possible.

Accomplished teachers attend to the primary goal—helping students develop sound mathematical knowledge, understanding, and ways of thinking. These teachers make necessary changes in accord with their best judgment about students' needs. They give students opportunities to develop expertise with the technology that will best help them learn good mathematics and solve

The Teaching of Mathematics

The ways a teacher makes decisions and implements plans in the classroom provide the most visible and—arguably—the most important demonstrations of accomplished practice. The next four standards describe the kinds of tasks teachers construct and select, the ways teachers facilitate classroom discourse, and the practices teachers use to assess and monitor learning. Accomplished mathematics teachers successfully perform these functions through the roles they assume, the organizational schemes they use, the decisions they make, and the ways they adjust their plans from moment to moment.

Standard VI: The Art of Teaching

Accomplished mathematics teachers stimulate and facilitate student learning by using a wide range of formats and procedures and by assuming a variety of roles to guide students' learning of mathematics.

Accomplished mathematics teachers value mathematics highly. They take joy in it. They are excited by the ideas they explore with students. And they communicate that joy to their students. For example, they share with students the remarkable fact that the ratio of the circumference to the diameter of any circle is always the same number and that the number π shows up in probability, statistics, number theory, and many other seemingly unrelated contexts.

Mathematics has the power to fascinate students—for example, the concept of infinity, the patterns found in Pascal's triangle, and the connection between Fibonacci numbers and the Golden Spiral. Accomplished teachers provide their students with opportunities to discover mathematical delights and to experience the intellectual satisfaction that comes from finding a solution to a problem or justifying a conjecture with a well-considered argument. They notice the light in the eyes of the students who are "turned on" to mathematics, and they seek ways to elicit the same excitement in those who have not yet been captivated.

Teachers modify classroom plans and activities in response to student needs, interests, and unexpected opportunities for learning. They demonstrate flexibility, insight, and responsiveness in dealing with the flow of the classroom. They recognize and respond to the mathematical potential of student questions and comments, and they pursue ideas of interest that emerge in classroom discussion. They also help students reflect on and extend their learning, and they expect them to take responsibility for their learning. There is no recipe for what teachers do at any given moment. Their choices are governed by their immediate and long-term goals, the progress and interests of their students, the instructional opportunities that present themselves, and the particular dynamics and tone of the day. However, there are important dimensions to accomplished teaching.

Teachers know that classroom interactions can develop a life of their own, that no plan should be followed simply for its own sake, and that they must adapt their plans where appropriate. They are prepared to adjust instruction—either because unforeseen diffi-

culties suggest that a path they had planned to take will not succeed, or because a classroom discussion points to a beneficial alternative. They are able to anticipate misunderstandings and provide instruction that will help as ideas unfold. Furthermore, they choose topics for discussion wisely, relying on their understanding of what is appropriate and important.

Accomplished teachers are willing to take risks in their classrooms. They vary standard practice, using conventional and unconventional methods to further their students' mathematical understandings. For example, they might help their students lead some aspect of instruction, or they might design a technology activity without first having all the answers themselves. They are not afraid to take calculated risks if they see better instructional opportunities in a new course of action. Their knowledge of mathematics and their understanding of different experiences help them move students toward important mathematical goals.

Accomplished teachers adjust the pace of the class. They moderate it to give students sufficient time to internalize concepts and build perspectives, to deepen and extend students' mathematical understanding, or to approach a new topic. They employ such teaching strategies as whole-class discussion, small-group work, individual study, and one-on-one sessions that allow students to explore, discover, and use mathematical ideas. They engage students in myriad activities, such as experiments, demonstrations, projects, games, puzzles and contests, writing, presentations, discussion, and debates. They make sound

judgments about the use of time and pacing, and they know when to adjust a classroom format to optimize learning.

Accomplished teachers assume different roles to accomplish their complex tasks, such as acting as a facilitator of student inquiry, an information provider, and a collaborator with students in solving problems. Accomplished teachers foster learning by choosing imaginative examples, problems, and situations designed to interest and motivate students, illuminate important ideas, or reveal the growth of student understanding. They work with small groups of students, asking clarifying or leading questions when necessary. They involve students in decisions about mathematical topics or ways to study those topics. They provide students with opportunities to reflect on their learning. And, they serve as a catalyst in launching student investigations.

Accomplished teachers promote meaningful discourse through the well-conceived questions they pose and through the rich tasks they provide. They demonstrate their use of appropriate questioning strategies by knowing how, when, and why to question students about their understanding of mathematics, and they provide a safe arena in which students can counter the arguments of others. They encourage students to pursue learning on their own.



Standard VII: Learning Environment

Accomplished mathematics teachers help students learn mathematics by creating environments in which students are active learners, show willingness to take intellectual risks, develop confidence and self-esteem, and value mathematics. This environment fosters students' learning of mathematics.

Accomplished mathematics teachers use their knowledge of how students learn to create a stimulating environment in which students are empowered to do mathematics and to foster a respectful, engaging, and cooperative atmosphere for learning. They encourage students to develop a good work ethic and to assume ownership and responsibility for the learning process.

Accomplished teachers create a positive learning environment. From the beginning of the school year, they engage their students in creating a community of learners in which students are willing to take intellectual risks. Such an environment is evident when teachers and students share ideas in a positive and productive manner; when students question mathematical ideas and concepts; when students realize that struggling and making errors are part of the process of learning; and when teachers invite students' discussion of mathematical activities. In such an environment of trust, students feel safe to communicate different points of view, to conduct open-ended explorations, to make mistakes and learn from them, and to admit confusion or uncertainty. Creating and maintaining such a learning environment require skill and planning, a variety of instructional methods, flexibility, good judgment, and discretion.

Accomplished teachers consider the mathematical understanding needs, interests,

and working styles of their students and the mathematics they are studying. They are sensitive to the needs of students with exceptionalities. They create a climate in which each student learns to value mathematics and experience success in doing significant mathematics. They lead by example, and they convey to students the delight that comes with the command of a mathematical tool or principle. They help students develop the ability to work both independently and collaboratively on mathematics.

Accomplished teachers encourage and capitalize on student discourse about mathematics. They invite students to question and offer conjectures, validate or refute assertions, offer proofs, and discuss and question their own thinking and the ideas proposed by others, allowing for natural acceptance of alternative approaches or solutions. They provide time for students to reflect on and assimilate the mathematics they are learning. They listen to what students say, look at students' work to gain insight into their understanding, and value and respect what students say and do.



Standard VIII: Ways of Thinking Mathematically

Accomplished mathematics teachers develop students' abilities to reason and think mathematically—to investigate and explore patterns, to discover structures and establish relationships, to formulate and solve problems, to justify and communicate their conclusions, and to question and extend those conclusions.

Mathematics is a discipline of concepts, principles, procedures, and reasoning processes. Its tools include representation, modeling, proof, experimentation, questioning, classification, visualization, and computation.⁶ Its practice has been profoundly affected and extended by technology—especially calculators and computers. In the classrooms of accomplished teachers, students are engaged in identifying patterns, solving problems, reasoning, forming and testing conjectures, and communicating results. They search for connections and solve problems while reflecting on both the mathematics and their own thought processes.

Accomplished teachers of mathematics recognize that important general concepts and reasoning methods undergird the development of mathematical power. They model mathematical reasoning as they work with students, and they encourage their students to question processes and challenge the validity of particular approaches. Their students make conjectures and justify or refute them, formulate convincing arguments, and draw logical conclusions. Sound reasoning—not an edict from the teacher—is the arbiter of mathematical correctness. In short, students become mathematically empowered as they learn to think, reason, and communicate mathematically.

Teachers recognize that mastering mathematical facts and procedures is only a part of what it means to learn mathematics. They also know the importance of developing students' understanding of and disposition to do mathe-

tics. They realize that teaching students to “think mathematically” means helping them develop a mathematical point of view; recognize situations in which mathematical reasoning might be useful; and have the ability, skill, and confidence to think through a situation. Consequently, they provide settings that allow students to test mathematical ideas, patterns, and conjectures; discover principles; synthesize evidence; and apply their growing knowledge to a variety of problems. Teachers know and use the overarching themes of mathematics that help students understand and appreciate the powerful relationships between mathematical ideas and problems, as in making students aware of the relationship between diverse fields, such as algebra and geometry or geometry and probability.

Teachers know multiple ways to represent mathematical ideas, and they organize tasks so that students will learn that a single problem may have many representations. Accomplished teachers encourage students to distinguish between these representations and to select a compelling and efficient representation for a given problem or situation. They teach students to recognize and seek alternative ways to solve mathematical problems. They provide opportunities for students to understand that multiple solutions can be informative, useful, and interesting. Mathematical reasoning permeates the tasks, actions, and discourse of accomplished teachers' mathematics classes.

Accomplished teachers provide students

6. Adapted from Mathematical Sciences Education Board, *On the Shoulders of Giants: New Approaches to Numeracy* (Washington, D.C.: National Academy Press, 1990), 3.

with problems and applications that will allow them to explore new mathematical content, to reflect on the problem-solving process, to extend and refine their thinking, and to make generalizations about the procedures they have used and link those generalizations with what they have learned previously.

Accomplished teachers provide many rich opportunities for students to apply mathematics to interesting problems. In doing so, they point out the interrelated domains of mathematics. They not only choose tasks related to everyday life—to the sciences, to economics, to politics, or to business—but they also choose tasks that will extend understanding within mathematics. Their choice of problem contexts reflects the breadth of mathematics and its applications.

Teachers also provide opportunities for students to recognize and formulate their own problems—problems that stem from their personal interests or experiences or that build on other work they are doing in mathematics.

Accomplished teachers deliberately structure opportunities for students to use and develop appropriate mathematical discourse as they reason and solve problems. These teachers give students opportunities to talk with one another and to work together in solving problems, and they have students use both written and oral discourse to describe and discuss their mathematical thinking and understanding. As students talk and write about mathematics—as they explain their thinking—they deepen their mathematical understanding in powerful ways that can enhance their ability to use the strategies and thought processes gained through the study of mathematics to deal with life issues.

Accomplished teachers are successful in using technology effectively to develop students' reasoning, mathematical thinking, and discourse. They are able to use applica-

tions such as graphing technology, interactive geometry software, and computer algebra systems not as “black boxes” that produce answers with little insight, but rather as tools for supporting student inquiry, conjecture, and proof.

Accomplished mathematics teachers encourage students to confront and challenge ideas and to question peers as they discuss mathematical ideas, develop mathematical understanding, and solve mathematical problems. They monitor what students do, using mathematical communication regularly to help students build understanding. They use probing and supportive questions to advance students' thinking about the use of available resources, the methodological choices they make to address problems, and the approaches they might pursue.

Accomplished teachers also tackle curriculum issues related to reasoning and mathematical thinking in their departments, schools, and districts. When appropriate, they make an effort to influence the way mathematics is taught at the lower grades. They inform themselves about how students develop reasoning ability. They consider carefully how different curriculum materials or issues might affect mathematical reasoning and thinking. For instance, they consider how an integrated mathematics curriculum might require special attention to the development of reasoning, or how scheduling strategies, such as block scheduling, might influence the emphasis on thinking or reasoning in different courses. Accomplished teachers are aware of the many influences on the process of learning and the ways curriculum decisions might have an intended or unintended impact on the instructional goals.



Standard IX: Assessment

Accomplished mathematics teachers employ a range of formal and informal assessment methods that are ongoing and embedded to evaluate student learning in light of well-defined goals. They employ multiple methods of assessment—including teacher-designed and external assessments, where appropriate—to diagnose learning; plan instruction; and provide opportunities for students to reflect on their strengths and weaknesses in order to revise, support, and extend their individual performance.

Accomplished teachers are guided by well-defined goals. They design appropriate assessment strategies and activities to monitor the progress of the class as a whole as well as the work of individual students. They consistently use a variety of methods to check for student understanding prior to, during, and at the close of a lesson. And they use this information to shape their teaching.

Teachers use a variety of assessment techniques that address various modalities and learning styles, including open-ended problems, group investigations, projects, portfolios, interviewing, and learning logs, to assess the processes as well as the products of students' mathematical explorations and problem-solving activities.

Teachers know that it is important for young adolescents to assume increasing responsibility for their own learning. They encourage students to set high goals, and they teach students how to evaluate their progress toward those goals. They give students frequent opportunities to use various performance measures as tools for self-reflection. They engage their students in critiquing and learning from their classmates' work, which can provide students with fresh perspectives on their own work. They also provide opportunities for students to reflect on and evaluate their progress and to revise, support, and extend their learning.

Teachers use formative assessment results to modify their lessons and the opportunities and activities they offer. They review or skip ahead to more challenging situations when appropriate. They use assessment strategies

to identify student strengths and areas for improvement, and they provide timely and instructive feedback.

As appropriate, teachers allow students to help formulate goals and criteria for success. They set high expectations and ensure that those expectations are clear, understandable, and understood by all their students. They establish clear criteria for success.

Besides assessing student learning using teacher-designed assessments, teachers are keenly aware of the external assessments that play an important part in a student's educational experience. District, state, and national tests and international studies all have significant impact on how the public perceives educational success and, in some cases, can have significant impact on a student's future. Teachers know the purposes and content of these external assessments. They evaluate their own curricular decisions in light of the content of these tests, ensuring that their students are well prepared for those examinations that will be important to their future goals. Teachers read and consider the curricular implications of data from national and international comparisons. They make an effort to influence the content and characteristics of these comparisons, particularly at the state and local levels. They use assessment as a basis to communicate and demonstrate that students are learning mathematics.



Professional Development and Outreach

Accomplished mathematics teachers, according to the Five Core Propositions, think systematically about their practice and learn from their experience. As professionals, they reflect on their own teaching and periodically adjust their course and instruction to respond to new ideas and the rate of progress they observe in their students.

But excellence and growth in the classroom are not enough in today's world. Today's teachers must show persistence and creativity in their efforts to involve families in meaningful ways. Today, more than ever, teachers must reach beyond their classrooms. They must participate in broader learning communities and reach out to the community at large. At the school, district, state, and national levels, teacher involvement is a key component in the effort to improve educational opportunities and learning for all students.

The definition of who and what a teacher is continues to evolve toward an emerging picture of a talented and competent professional who is charged with the most important function in society—educating tomorrow's adults. Important signs of this professionalism are reflected in the final cluster of standards.

Standard X: Reflection and Growth

Accomplished mathematics teachers regularly reflect on what they teach and how they teach. They keep abreast of changes and learn new content in mathematics and in mathematical pedagogy, continually seeking to improve their knowledge and practice.

For accomplished mathematics teachers, every class and every course provide the opportunity to reflect and improve. When things go well, accomplished teachers think about how to apply the lessons learned to other situations. When things do not go well, they consider how to improve.

This would be the case even if the world of mathematics were static. However, the body of mathematics itself and the pedagogical bases for teaching mathematics are constantly evolving. The knowledge base of mathematics is growing, and new mathematical fields and technology are becoming increasingly important.

Researchers have learned a great deal about how people think about and learn mathematics. Accomplished teachers stay

current with professional research and regularly refine their practices in light of both professional knowledge and personal experience. Even the most effective mathematics teachers regularly refine their practice.

Accomplished teachers keep abreast of changes in mathematics and mathematical pedagogy through such activities as reading professional journals, dialoguing with peers, attending meetings and conferences, and participating in professional organizations. They continually participate in professional development. Whether extending their formal education or engaging in a self-directed plan, these teachers actively pursue ways of enhancing their own knowledge and skills. They identify areas for self-improvement and seek strategies for reaching their educational goals.

Standard XI: Families and Communities

Accomplished mathematics teachers support and promote the involvement of families in their children's education. They help varied communities understand the role of mathematics and mathematics instruction in today's world, and—to the extent possible—they involve these communities in the support of instruction.

Accomplished mathematics teachers recognize the complexity of the home-school-student relationship, and they view families as partners who can encourage children to appreciate and value mathematics. They respect the role of family members as children's first teachers and as supporters of children's growth and development. They look to families for information about children's strengths, interests, dispositions, habits, and home life.

Accomplished teachers help family members learn about key components and issues of school programs, the significance of test scores and grades, the consequences of taking or not taking certain courses, the reasons for group and class assignments, and the benefits of planning for future education. They help families understand mathematics methodologies other than the ones that they were taught in school. They often design homework assignments that will encourage family discussion of school subjects. However, these teachers are mindful that not all students have families to assist them, and they suggest appropriate options for such students. Accomplished teachers work with families to help their children develop good learning habits and study skills, complete homework, set goals, and improve performance. Recognizing the potential benefits to learning, they invite family members to participate in their children's educational experiences.

Accomplished teachers provide support and encouragement when students are doing well and mobilize efforts to help students

who are having difficulty. They strive to include family members as educational partners, they advise family members of student progress on a regular basis, and they respond to family concerns. They help family members know the short- and long-term goals and objectives of mathematics instruction, and they inform them about available materials and programs that will broaden their students' mathematical experiences.

Accomplished teachers realize that the best support for good mathematics instruction often comes from informed family members. They recognize the difficulties inherent in engaging families at a time when their children are striving for independence as young adults. These teachers also create activities designed to help families recognize the connections between mathematics education and their children's future lives. These might include creating a welcoming environment that encourages family members to visit the school and learn about mathematics education; working with family advisory groups; supporting mathematics fairs and other events that allow students to demonstrate their mathematical skills; involving family members in career days; and helping students explore career opportunities.

Accomplished teachers work proactively to help students, families, and the community at large understand the role of mathematics and mathematics instruction. They reach beyond the families of their students to the community at large; help community members become aware of, supportive of, and involved in the school mathematics

Standard XII: Contributing to the Professional Community

Accomplished mathematics teachers collaborate with peers and other education professionals to strengthen their school's programs, advance knowledge, and contribute to improving practice within the field.

Accomplished mathematics teachers do not work in isolation; they are members of learning communities. They contribute to the quality of practice of their peers and to the design of the community's overall instructional program. They collaborate with parents, colleagues, and other members of the community in the education of adolescents and young adults.

Seeing themselves as partners with other teachers, accomplished teachers are dedicated to improving the profession. They care about the quality of teaching in their schools, and—to this end—their collaboration with colleagues is continuous and explicit. For example, they may work with preservice teachers, serve as mentors to new teachers, or participate as peer coaches for experienced teachers.

Accomplished teachers work effectively with other school professionals to make students feel part of a community with a coherent ethos and to help them integrate knowledge across the disciplines in areas of applications, connections, and extensions. For example, they may work with language arts teachers to share ideas on communication skills and writing in mathematics; with social studies teachers on incorporating problem solving or data analysis; with visual arts teachers on spatial relationships and graphic representation; and with science teachers on investigations and analyses.

Since accomplished teachers are dedicated to their students and to their learning of

mathematics, the focus of their contribution to the larger learning community often revolves around curricular or assessment issues. These may include periodically reviewing district and school curricula to help ensure that they are thoughtfully organized, consistent with the profession's views of best practices, and tailored to the students and the community they serve; reviewing external testing instruments and practices in their school or district and analyzing their alignment with the goals of the mathematical community and the mathematical goals of the school and the classroom; and serving as advocates for the appropriate use of sound internal assessment instruments and quality curriculum.

As an extension of their local responsibilities, accomplished teachers often engage in a wide range of professional activities. They make productive contributions to the advancement of their field through active participation in professional organizations at the local, state, regional, and national levels. They work to support the improvement of mathematics education and the professional growth of mathematics teachers at all levels. Such commitment is central to their dedication to the quality of their practice and to the advancement of mathematics education.



The 12 standards in this report represent a professional consensus on the characteristics of accomplished mathematics practice and provide a profile of the accomplished Adolescence and Young Adulthood/Mathematics teacher. Although the standards are challenging, they are upheld every day by teachers like the ones described in these pages, who inspire and instruct the nation's youth and lead their profession. By publishing this document and offering National Board Certification to mathematics educators, NBPTS aims to affirm the practice of the many teachers who meet these standards and challenge others to strive to meet them. Moreover, NBPTS hopes to bring increased attention to the professionalism and expertise of accomplished mathematics educators and in so doing, pave the way for greater professional respect and opportunity for these essential members of the teaching community.

In addition to being a stimulus for self-reflection on the part of teachers at all levels of performance, *Adolescence and Young Adulthood/Mathematics Standards* is intended to be a catalyst for discussion among administrators, staff developers, and others in the education community about accomplished practice in this field. If these standards can advance the conversation about accomplished teaching, they will provide an important step toward the NBPTS goal of improving student learning in our nation's schools.

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The National Board for Professional Teaching Standards' *Adolescence and Young Adulthood/Mathematics Standards, Second Edition*, reflects more than a decade of dialogue about accomplished teaching in mathematics. These standards derive their power from an amazing degree of collaboration and consensus. Through the expertise and input of two standards committees, convened six years apart; numerous reviews by a 63-member board of directors; and two periods of public comment by educators, policymakers, parents, and the like; as well as through the intense study of candidates for National Board Certification who have immersed themselves in the first edition; these second-edition standards emerge as a living testament to what accomplished teachers should know and be able to do. *Adolescence and Young Adulthood/Mathematics Standards, Second Edition*, represents the best thinking by teachers and for teachers about advanced teaching practice in the field.

The National Board for Professional Teaching Standards is deeply grateful to all of those who contributed their time, wisdom, and professional vision to *Adolescence and Young Adulthood/Mathematics Standards, Second Edition*. Any thank-you must begin with the pioneers in 1990 who spent six years debating, reflecting, and articulating the multiple facets of accomplished teaching, so that they could help advance the field and also provide a rigorous and sound basis for national certification of teachers. In particular, the National Board would like to show its appreciation to Chair Gail Burrill and Vice Chair John Dossey, who so skillfully led the effort to weave the National Board's Five Core Propositions into field-specific standards of teaching excellence.

Any field grows, shifts, and evolves over time. Standards, too, must remain dynamic and therefore are subject to revision. In January 2000, the National Board for Professional Teaching Standards convened a second Adolescence and Young Adulthood/Mathematics Standards Committee. This committee was charged with achieving both continuity and change, using the first edition of the standards as the foundation for its work, but modifying the standards to reflect best practice of the early twenty-first century. The Adolescence and Young Adulthood/Mathematics Standards Committee exemplified the collegiality, expertise, and dedication to the improvement of student learning that are hallmarks of accomplished teachers. Special thanks go to Chair Carol Malloy and Vice Chair Cliff Barrineau for their invaluable leadership in making the second edition a reality.

The Standards and Professional Development Working Group of the board of directors is also an important collaborator in the creation of the second-edition standards. The working group consists of a diverse group of educators who reviewed *Adolescence and Young Adulthood/Mathematics Standards, Second Edition*, at various points in its development, made suggestions about how it could be strengthened, and recommended to the full board adoption of the standards. Representing the board of directors as a liaison to the Adolescence and Young Adulthood/Mathematics Standards Committee was Larry Peterson, whose extensive knowledge of the field made him a treasured advisor.

Hundreds of individuals not directly associated with the National Board aided in the development of these standards. Mathematics teachers and scholars, state and local officials, and representatives of disciplinary organizations—to name just a few—reviewed a draft of *Adolescence and Young Adulthood/Mathematics Standards, Second Edition*, when the standards were disseminated nationwide during a public comment period.

Acknowledgments

Many staff members of the National Board also deserve thanks for helping to make the publication of these standards possible. Chuck Cascio, former Vice President for Certification Standards and Teacher Development, shepherded the standards from their inception. In the early stages, Jacqueline Olkin, former Manager for Certification Standards and Teacher Development, was especially instrumental. Writing credits go to Carol Bruce, consultant to the National Board; Angela Duperrouzel served as on-site coordinator for standards committee meetings; Holly Baker edited the document during production. I would like to give a special thanks to the dedicated staff I have worked with: Michael Knab, Manager for Certification Standards; Teachers-in-Residence Mary Lease, NBCT, and Maria Telesca, NBCT; Jane George, Specialist for Certification Standards Production; and Administrative Assistant Glowena Harrison. National Board staff collaborated in all aspects of standards development.

In presenting these standards for accomplished teaching, the National Board for Professional Teaching Standards recognizes that this publication would not have been possible without the considerable contributions of individuals and institutions too numerous to mention. On behalf of the National Board for Professional Teaching Standards, I extend my thanks to all of them.

Katherine S. Woodward
Director, Certification Standards



The core propositions of the National Board for Professional Teaching Standards

- 1) *Teachers are committed to students and their learning.*
- 2) *Teachers know the subjects they teach and how to teach those subjects to students.*
- 3) *Teachers are responsible for managing and monitoring student learning.*
- 4) *Teachers think systematically about their practice and learn from experience.*
- 5) *Teachers are members of learning communities.*