

# Middle Childhood through Early Adolescence/Mathematics **STANDARDS**

(for teachers of students ages 7–15)



National Board



***T**he National Board would like to express appreciation to The Pew Charitable Trusts and the U.S. Department of Education for their support of the cost of developing this standards report, and to the DeWitt Wallace-Reader's Digest Fund for its publication.*

***T**his project is funded in part with grants from the U.S. Department of Education and the National Science Foundation. Through September 1997, NBPTS has received federal funds of \$35 million, representing approximately 41 percent of the National Board Certification project. More than \$51 million (59 percent) of the project's cost has been financed by nongovernmental sources.*

# Table of Contents

---

(for teachers of students ages 7–15)

<b>Preface</b> .....	<b>1</b>
<b>Introduction</b> .....	<b>7</b>
<b>Overview</b> .....	<b>11</b>
<b>The Standards</b> .....	<b>13</b>
<b>I</b> Commitment to Equity and Access .....	13
<b>II</b> Knowledge of Students .....	15
<b>III</b> Knowledge of Mathematics .....	19
<b>IV</b> Knowledge of Teaching Practice .....	25
<b>V</b> The Art of Teaching .....	29
<b>VI</b> Learning Environment .....	33
<b>VII</b> Using Mathematics .....	35
<b>VIII</b> Technology and Instructional Resources .....	39
<b>IX</b> Assessment .....	41
<b>X</b> Reflection and Growth .....	43
<b>XI</b> Families and Communities .....	45
<b>XII</b> Professional Community .....	47
<b>Epilogue</b> .....	<b>49</b>
<b>Standards Committee</b> .....	<b>51</b>
<b>Acknowledgements</b> .....	<b>53</b>

---

The world-class schools America requires cannot exist without a world-class teaching force; the two go hand in hand. Many accomplished teachers already work in the nation's schools, but their work is often unrewarded and underappreciated, their knowledge and skills unacknowledged and underutilized. Delineating accomplished practice and recognizing those who achieve it are important first steps in shaping the kind of teaching profession America needs. This is the core challenge embraced by the National Board for Professional Teaching Standards (NBPTS). Founded in 1987 with a broad base of support from governors, teacher union and school board leaders, administrators, college and university officials, business executives, foundations, and concerned citizens, NBPTS is a nonprofit, nonpartisan organization governed by a 63-member Board of Directors, the majority of whom are teachers. Committed to basic reform in American education, the National Board recognizes that teaching is at the heart of education and, further, that the single most important action the nation can take to improve schools is to strengthen teaching. To this end, the National Board has embarked on a three-part mission:

- to establish high and rigorous standards for what accomplished teachers should know and be able to do;
- to develop and operate a national voluntary system to assess and certify teachers who meet these standards; and
- to advance related education reforms for the purpose of improving student learning in American schools.

Achieving this mission will elevate the teaching profession, educate the public about the demands and complexity of accomplished practice, and increase the chances of attracting and retaining in the profession talented college graduates with many other promising career options.

National Board Certification is more than a system for recognizing and rewarding accomplished teachers, however. It represents both an opportunity to rethink the way the profession organizes itself for the continuing growth and development of its members and a chance to design new ways to organize and manage schools so as to capitalize on the expertise of accomplished teachers. Together with other reforms, it can be a catalyst for significant change in the profession and in American education.

## The Philosophical Context

The standards presented here lay the foundation for the Middle Childhood through Early Adolescence/Mathematics certificate. They represent a professional consensus on the critical aspects of practice that distinguish accomplished teachers in this field. Cast in terms of actions that teachers take to advance student outcomes, these standards also incorporate the essential knowledge, skills, dispositions, and commitments that allow teachers to practice at a high level. Like all National Board standards, they rest on a fundamental philosophical foundation expressed in the NBPTS policy statement *What Teachers Should Know and Be Able to Do*. That statement identifies five core propositions:

### 1) Teachers are committed to students and their learning.

Accomplished teachers are dedicated to making knowledge accessible to all students. They act on the belief that all students can learn. They treat students equitably, recognizing the individual differences that distinguish their students one from the other and taking account of these differences in their practice. They adjust their practice, as appropriate, on

the basis of observation and knowledge of their students' interests, abilities, skills, knowledge, family circumstances, and peer relationships.

Accomplished teachers understand how students develop and learn. They incorporate the prevailing theories of cognition and intelligence in their practice. They are aware of the influence of context and culture on behavior. They develop students' cognitive capacity and their respect for learning. Equally important, they foster students' self-esteem, motivation, character, civic responsibility, and respect for individual, cultural, religious, and racial differences.

## **2) Teachers know the subjects they teach and how to teach those subjects to students.**

Accomplished teachers have a rich understanding of the subject(s) they teach and appreciate how knowledge in their subject is created, organized, linked to other disciplines, and applied to real-world settings. While faithfully representing the collective wisdom of our culture and upholding the value of disciplinary knowledge, they also develop the critical and analytical capacities of their students.

Accomplished teachers command specialized knowledge of how to convey and reveal subject matter to students. They are aware of the preconceptions and background knowledge that students typically bring to each subject and of strategies and instructional materials that can be of assistance. They understand where difficulties are likely to arise and modify their practice accordingly. Their instructional repertoire allows them to create multiple paths to the subjects they teach, and they are adept at teaching students how to pose and solve their own problems.

## **3) Teachers are responsible for managing and monitoring student learning.**

Accomplished teachers create, enrich, maintain, and alter instructional settings to capture and sustain the interest of their students and to make the most effective use of time. They are also adept at engaging students and adults to assist their teaching and at enlisting their colleagues' knowledge and expertise to complement their own.

Accomplished teachers command a range of generic instructional techniques, know when each is appropriate, and can implement them as needed. They are as aware of ineffectual or damaging practice as they are devoted to quality practice.

They know how to engage groups of students to ensure a disciplined learning environment, and how to organize instruction to allow the schools' goals for students to be met. They are adept at setting norms for social interaction among students and between students and teachers. They understand how to motivate students to learn and how to maintain their interest even in the face of temporary failure.

Accomplished teachers can assess the progress of individual students as well as that of the class as a whole. They employ multiple methods for measuring student growth and understanding and can clearly explain student performance to parents.

## **4) Teachers think systematically about their practice and learn from experience.**

Accomplished teachers are models of educated persons, exemplifying the virtues they seek to inspire in students—curiosity, tolerance, honesty, fairness, respect for diversity and appreciation of cultural differences—and the capacities that are prerequisites for

intellectual growth: the ability to reason and take multiple perspectives, to be creative and take risks, and to adopt an experimental and problem-solving orientation.

Accomplished teachers draw on their knowledge of human development, subject matter, and instruction and their understanding of their students to make principled judgments about sound practice. Their decisions are not only grounded in the literature of their field, but also in their experience. They engage in lifelong learning, which they seek to encourage in their students.

Striving to strengthen their teaching, accomplished teachers examine their practice critically, and seek to expand their repertoire, deepen their knowledge, sharpen their judgment, and adapt their teaching to new findings, ideas, and theories.

## 5) Teachers are members of learning communities.

Accomplished teachers contribute to the effectiveness of the school by working collaboratively with other professionals on instructional policy, curriculum development, and staff development. They can evaluate school progress and the allocation of school resources in light of their understanding of state and local educational objectives. They are knowledgeable about specialized school and community resources that can be engaged for their students' benefit and are skilled at employing such resources as needed.

Accomplished teachers find ways to work collaboratively and creatively with parents, engaging them productively in the work of the school.

## The Certification Framework

Using the five core principles as a springboard, NBPTS will set standards and award certificates in nearly 30 fields. Most of these fields are delineated by two factors, the developmental level of the students and the subject or subjects being taught. The first dimension embraces four overlapping student development levels:

- Early Childhood, ages 3–8
- Middle Childhood, ages 7–12
- Early Adolescence, ages 11–15
- Adolescence and Young Adulthood, ages 14–18+.

The second dimension highlights the substantive focus of a teacher's practice, allowing most teachers to select either a subject-specific or a generalist certificate. At the early adolescence level, for example, teachers will be able to pursue either a generalist certificate or a certificate in English language arts, mathematics, science, or social studies–history. Teachers seeking certification at the middle childhood level will have a similar choice. The framework of certificates will also include a generalist certificate at the early childhood level and subject-specific certificates at the adolescence and young adulthood level. In some fields, developmental levels are joined together (for example, Early Adolescence through Young Adulthood/Vocational Education) to recognize the commonalities in practice at those levels.

A third dimension comes into play in two other areas: the special knowledge associated with teaching children with exceptional needs in the first instance, and the special knowledge associated with teaching those for whom English is a new language in the second.

## **Standards and Assessment Development**

Standards committees are appointed for each of these certificate fields. The committees are generally made up of 15 members who are broadly representative of accomplished professionals in their field, and a majority of whom are teachers regularly engaged in practicing with students in the field in question. Other members are typically experts in child development, teacher education, and the relevant discipline(s). The standards committees recommend to the National Board the specific standards for each field, which are then disseminated widely for public critique and comment and subsequently revised as necessary.

Determining whether candidates meet the standards requires performance-based assessment methods that are fair, valid, and reliable and that call on teachers to demonstrate principled professional judgments in a variety of situations. A general contractor for assessment development, working with standards committee members, develops assessment exercises and pilot tests them with small groups of teachers. These education measurement experts and educators have deep roots in the certification field. The assessment process is structured around two key activities: (1) the compilation of a teacher's portfolio of practice during the course of a school year; and (2) participation in one day of assessment center activities during the summer.

Teachers prepare their portfolios by gathering student learning products and other teaching artifacts and providing analyses of their practice. At the assessment center teachers take part in exercises organized around challenging issues that teachers in their field face on a regular basis. The portfolio is designed to capture teaching in real-time, real-life settings, thus allowing NBPTS to examine how teachers translate knowledge and theory into practice. It also yields the most valued evidence NBPTS collects—videos of practice and samples of student work. The videos and student work are accompanied by commentaries on the goals and purposes of instruction, the effectiveness of the practice, teachers' reflections on what occurred, and their rationales for the professional judgments they have made. In addition, the portfolio allows candidates to document their accomplishments in contributing to the advancement of the profession and the improvement of schooling at the local, state, or national level, and to document their ability to work constructively with their students' families.

Teachers report that the portfolio is a professional development vehicle of considerable power, in part because it challenges the historic isolation of teachers from their peers. It accomplishes this by actively encouraging candidates as they build their portfolios to seek the advice and counsel of their professional colleagues from across the hall or across the country. It also requires teachers to examine the underlying assumptions of their practice and the results of their efforts. This emphasis on reflection is highly valued by teachers who go through the process.

The assessment center exercises are designed to complement the portfolio, validate that the knowledge and skills exhibited in the portfolio are, in fact, accurate reflections of what candidates know and can do, and give candidates an opportunity to demonstrate knowledge and skills not sampled in the portfolio because of the candidate's specific teaching assignment. For example, high school mathematics teachers assigned to teach only algebra in a given year might be unable to demonstrate in their portfolio broad knowledge of geometry. Given that NBPTS' standards for mathematics teachers place a high value on broad knowledge of the various content strands within mathematics at each developmental level, another strategy for data collection is necessary. The assessment center exercises are designed to ascertain additional subject-matter knowledge and

otherwise augment the portfolio. The exercises sample the breadth of the content knowledge and pedagogy associated with the certificate field through authentic scenarios that allow candidates to confront important instructional matters even as they are removed from the immediacy of the classroom. Each candidate's work is examined by assessors who are themselves accomplished teachers in the certificate field.

The National Board for Professional Teaching Standards believes a valid assessment of accomplished practice must allow for the variety of forms sound practice takes, must sample from the range of ways of knowing required for teaching, and must place assessments of teaching knowledge and skill in appropriate contexts. Teaching is not just about knowing things; it is about the use of knowledge—knowledge of learners and learning, of schools and subjects—in ways that help students grow and develop. Consequently, NBPTS believes that the most valid teacher assessment processes engage candidates in the activities of teaching—activities that require the display and use of teaching knowledge and skill and that allow teachers the opportunity to explain and justify their actions.

In addition, in its assessment development work, NBPTS explores and uses, when appropriate, state-of-the-art technology for assessment; ensures broad representation of the diversity that exists within the profession in all stages of the development process; engages the pertinent disciplinary and specialty associations at key points in the process; works closely with appropriate state agencies, academic institutions, and independent research and education organizations; establishes procedures to detect and eliminate instances of external and internal bias with respect to age, gender, and racial and ethnic background of teacher candidates; and selects the method exhibiting the least adverse impact when given a choice among equally valid assessments.

Once a certificate has been thoroughly tested and found to meet the National Board's requirements for validity, reliability, and fairness, eligible teachers may apply for National Board Certification. To be eligible, a teacher must hold a baccalaureate degree from an accredited institution; have a minimum of three years of teaching experience at the early childhood, elementary, middle school, or high school level; and, where it is required, hold a state teaching license.

## **Strengthening Teaching and Improving Learning**

A system of National Board Certification that commands the respect of the profession and the public can make a critical difference in how communities and policy makers view teachers, in how teachers view themselves, and in how teachers improve their practice throughout their careers. National Board Certification has the potential to yield such significant results for American education in part because it marks the first attempt to forge a national professional consensus on the critical aspects of accomplished practice in each teaching field. The traditional conversation about teacher competence has focused on beginning teachers. Yet, unless we believe that the professional development of teachers should conclude the day novice teachers are hired, this new conversation about accomplished practice is essential.

Developing standards of accomplished practice has the potential to lift the entire profession as the standards make public the knowledge, skills, and dispositions that set accomplished teachers apart from journeymen. However, converting such standards into a system for the advanced certification of teachers promises much more. A mechanism that can identify accomplished teachers in a fair and trustworthy manner can accelerate efforts to build school organizations, structures, and career paths that look significantly different from the flat, undifferentiated approach that typically gives novice teachers

much more responsibility than they can sensibly handle, fails to take best advantage of the knowledge, wisdom, and expertise of accomplished teachers, and encourages many accomplished practitioners to leave the classroom for greater status, authority, and compensation.

By holding accomplished teachers to high and rigorous standards, National Board Certification has the potential to leverage change along several key fronts. For example:

- changing what it means to have a career in teaching by recognizing and rewarding accomplished teachers and by making it possible for teachers to advance in responsibility, status, and compensation without having to leave the classroom;
- changing the culture of teaching by accelerating growth in the knowledge base of teaching, placing real value on professional judgment and accomplished practice in all its various manifestations, and encouraging among teachers the search for new knowledge and better practice through collaboration and reflection with peers and others;
- changing the way schools are organized and managed by creating a vehicle that facilitates the establishment of lead teacher positions and thus provides accomplished teachers with greater authority and autonomy in the making of instructional decisions and greater responsibility for sharing their expertise to strengthen the practice of others;
- changing the nature of teacher preparation and ongoing professional development by laying a standards-based foundation for a fully articulated career-development path that begins with prospective teachers and continues from novice through accomplished teachers;
- changing the way school districts think about hiring and compensating teachers by encouraging administrators and school boards to search for accomplished teachers, rather than those who can be hired at the lowest salary, and to reward excellence in teaching.

Although National Board Certification has been designed with the entire country in mind, each state and locality will decide for itself how best to encourage teachers to stand for advanced certification and how best to take advantage of the expertise of the National Board Certified Teachers in their midst. One of the many examples of state action in support of National Board Certification comes from North Carolina, where legislation has been enacted that provides funds to pay the certification fee for teachers who complete the process. This legislation also funds release time for candidates to work on their portfolios and prepare for the assessment center exercises and provides salary increases for teachers who achieve National Board Certification. Other states have adopted or are in the process of adopting legislative initiatives that acknowledge National Board Certification and offer incentives for certification. At the local level, to give just two examples, school districts in Boston, Massachusetts, and Rochester, New York, have agreed to absorb the fee for the assessment process and give National Board Certified Teachers consideration for special assignments.

As this growing support at the state and local level suggests, National Board Certification is being recognized for the rich professional-development experience it has proven to be. In addition, it provides states and localities opportunities to structure teachers' roles and responsibilities and organize schools in ways that make better use of the strongest teachers. Therefore National Board Certification can combine with other initiatives to make dramatic improvements in American education.

A look into a mathematics classroom in the 1990s can present a picture very different from that of mathematics classrooms over the previous 100 years, or even the previous ten years. In preparing students for the exciting and increasingly challenging mathematics of high school and beyond, teachers of mathematics for children in the middle-childhood-through-early-adolescence range have a more difficult task than ever, making twenty-first century mathematics accessible to all students. Accomplished mathematics teachers today incorporate new and innovative approaches to foster student learning. Gone are the tedious hours of drilling formerly seen as the only way to memorize a narrow set of skills; the notion that mathematics comes packaged in neat, self-contained compartments; the belief that all problems have predictable answers; the reliance on a single way of solving a category of problems; and the attitude that only the privileged or gifted can understand and do mathematics. Whereas helping students attain computational proficiency remains a critical part of mathematics teaching, greater emphasis is now placed on problem solving in the context of real-life situations and on the interconnected and complementary nature of the various strands of mathematics. For example, students today might use technology to analyze numbers in an election, telecommunications to retrieve statistical data from satellites in order to analyze weather patterns, or geometry to design cafeteria facilities to maximize seating capacity. Perhaps the most important difference in today's mathematics classroom is the clear evidence that every student can engage in and succeed at significant, challenging, relevant mathematics.

Against this backdrop, the teacher of mathematics at the middle childhood through early adolescence level plays a critical role that is continually evolving as more is learned about students and mathematics. The mathematics teacher is a lens through which students see the world of mathematics, a creator of learning opportunities, and an advocate for each student in the mathematics classroom, opening doors to a wide range of learning possibilities and future options. Today's teacher of middle childhood or early adolescence mathematics can be found in an ungraded primary school, in a self-contained elementary classroom, in a departmentalized intermediate school, on an interdisciplinary middle school team, in a one-room schoolhouse, or in a traditional junior high school. Yet, regardless of the school structure, every mathematics teacher can find opportunities for creativity, academic challenge, and personal fulfillment. And every mathematics teacher faces new roles and responsibilities in performing the job of teacher in a setting that may or may not match his or her professional education experiences or experience as a student.

In the past decade, there have been significant changes in mathematics as a discipline, and the role mathematics plays in educational and economic opportunity has become increasingly important. The development of modern technology has profoundly changed the ways in which organizations and individuals all over the world achieve their objectives. Technological advances have substantially broadened the applications of mathematics, not only in education, science, industry, business, and government, but also more prominently than before in areas such as agriculture, health, social sciences, politics, and the arts. The changing nature of work in the United States, coupled with stiff international competition, requires that all citizens, not just a select few, become mathematically literate. The concepts and principles every citizen must acquire to participate fully in the modern world are more complex than those required of earlier generations. Young adult Americans must master mathematical ways of thinking if they are to succeed and if they are to make informed decisions about issues critical to their lives and to society.

Several national reports highlight these transforming events and new realities and provide recommendations for the reform of school mathematics and its teaching.<sup>1</sup> Each of these reports asserts the importance of preparing students who will mature into adults with confidence and competence in their mathematical knowledge and abilities.

1 Mathematical Association of America, *A Call for Change*. Washington, DC: Author, 1991.

Mathematical Sciences Education Board, *Everybody Counts*. Washington, DC: National Academy Press, 1989.

National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: Author, 1989.

National Council of Teachers of Mathematics, *Professional Standards for Teaching Mathematics*. Reston, VA: Author, 1991.

This conception of mathematics teaching is predicated on the belief that all students, regardless of their academic and economic background, gender, race, and ethnicity, must be enabled to achieve the level of mathematical competence required to succeed in life and in work. It recognizes not only that the use of mathematics as a gatekeeper must be discarded, but also that the capacity of mathematics to empower young people must be embraced. The ongoing changes in mathematics and the profession's emerging vision of mathematics instruction place high expectations on, and demanding challenges for, the nation's teachers.

The National Board recognizes that over the past decade, through an extensive process of debate, discussion, and self-criticism, the mathematics community has articulated a rich and compelling vision for teaching school mathematics. The Middle Childhood through Early Adolescence/Mathematics standards committee has embraced this conception of mathematics instruction in the standards it has defined. At the same time, it believes the assessment of candidates for National Board Certification against these standards must be tempered by the realization that the incorporation of these principles into teachers' daily practice will occur gradually.

National Board Certification must be built on high and rigorous standards and trustworthy assessment processes, but it must also be grounded in a sense of fair play for all candidates and respect for the diverse circumstances in which teachers find themselves. Periodically, such standards need to be reevaluated to reflect new understandings of how children learn and new knowledge about teaching practice. Consequently, it is reasonable to expect that what may be considered highly innovative practice in the early years of National Board Certification will, in time, be incorporated into the standards as reasonable expectations for accomplished teachers.

The standards committee has proposed a challenging set of requirements for National Board Certification and expects that successful candidates will meet each standard, demonstrating their accomplished practice in a variety of ways. Because there is no single best model of accomplished teaching to fit all contexts, assessment must be flexible enough to accommodate multiple profiles of accomplished practice.

Many accomplished teachers already meet or exceed the challenges presented in these standards. Others entering the teaching profession continue to join these teachers in the work needed to achieve the vision of the National Council of Teachers of Mathematics (NCTM) Standards. The Carnegie Task Force on Teaching as a Profession<sup>2</sup> had a similar vision when it articulated the idea of publicly certifying the work of exceptional teachers who make a difference in the lives and abilities of the children they teach. The Carnegie report stimulated the formation of the National Board for Professional Teaching Standards, which in turn led to the development of the standards in this report—standards intended to recognize mathematics teachers who tap the nation's mathematical talent and ensure the progress of all students toward high achievement in mathematics.

2 Carnegie Task Force on Teaching as a Profession. *A Nation Prepared: Teachers for the 21st Century*. Washington, DC: Author, 1986.

## The Standards Format

The main task of the Middle Childhood through Early Adolescence/Mathematics standards committee, then, has been to take the National Board's five propositions to the next level of specificity—to describe what they mean in terms of accomplished practice in each of these two fields.

One of the essential tensions of carrying out this assignment concerns the difference between the analysis and the practice of teaching. The analysis of teaching tends to fragment the profession into any number of discrete duties—designing learning activities, managing the classroom, monitoring student progress, and so on. Teaching as it actually occurs, on the

other hand, is a seamless activity. Everything the accomplished mathematics teacher knows through study, research, and experience is brought to bear daily in the classroom in the form of, literally, dozens of decisions that shape the learning environment. These judgments range from the tactical to the strategic, frequently require balancing the competing claims of several important instructional goals, draw on observations of particular students and settings, and are subject to revision on the basis of continuing developments in the classroom. In the real world, such functions as planning learning activities, motivating effort, assessing progress, and maintaining classroom discipline do not happen in isolation. Rather, they are skillfully interwoven strands in the single, sturdy fabric of quality practice.

The paradox, then, is that any attempt to write standards that take apart what accomplished teachers know and are able to do will, to a certain extent, also compromise the holistic nature of how teaching actually takes place. Nevertheless, the fact remains: Certain identifiable commonalities characterize the many styles of accomplished practice in teaching children mathematics. These “big ideas” are the focus of this report; they have been laid out in the form of twelve standards, the description of which makes up the heart of the document.

These specific standards are not, however, the only way the task of describing accomplished mathematics teaching could have been approached. Nor is each of equal weight. No linearity, atomization, or hierarchy in this vision of accomplished teaching is implied. Instead, the standards are presented as aspects of mathematics teaching that are analytically separable for the purposes of this document but that are not discrete in practice.

The format of the report follows a threefold approach to describing the twelve standards:

- I. **Definition of the Standard**—This is a succinct statement of one aspect of the practice of accomplished Middle Childhood through Early Adolescence/Mathematics teachers. Each of the standards is couched in terms of observable teacher actions that have an impact on students.
- II. **Elaboration**—This provides “texture” for the standard, along with an explanation of what teachers need to know and value if they are to meet the standard at a high level. It includes descriptions of their orientation to students, their distinctive roles and responsibilities, and their stances on a range of ethical and intellectual challenges that regularly confront them.

The National Board does not believe there is a single correct way to teach children. The examples presented here are but a handful of those that could have been written to describe the many approaches used in accomplished Middle Childhood through Early Adolescence/Mathematics teaching practice.

Finally, a word about the order of presentation: The twelve standards have been organized around the critical nexus of education—student learning. They are divided into three categories: 1) teacher actions that create the conditions for productive student learning; 2) teacher actions that directly advance student learning in the classroom; and 3) teacher actions that indirectly support student learning through professional development and outreach initiatives.



# Middle Childhood through Early Adolescence/Mathematics

(for teachers of students ages 7–15)

## OVERVIEW

The Middle Childhood through Early Adolescence/Mathematics Standards Committee recommends the following twelve standards for National Board Certification in each of these two fields. The standards have been sequenced to facilitate understanding, not to

assign priorities. In fact, in the course of quality mathematics teaching, accomplished teachers often demonstrate several of these standards concurrently as they skillfully weave their knowledge, skills, and dispositions into a rich tapestry of accomplished practice.

### Creating a Productive Learning Environment

#### Commitment to All Students

---

##### I. Commitment to Equity and Access (p. 13)

Accomplished mathematics teachers value and acknowledge the individuality and worth of each student; they believe that all students can learn and should have access to the full mathematics curriculum; and they demonstrate these beliefs in their practice by systematically providing all students equitable and complete access to mathematics.

#### Knowledge of Students, Mathematics, and Teaching

---

##### II. Knowledge of Students (p. 15)

Accomplished mathematics teachers recognize that students are shaped by a variety of educational, social, and cultural backgrounds and experiences that influence learning. They draw on knowledge of how students learn and develop in order to understand students and to guide curricular and instructional decisions.

##### III. Knowledge of Mathematics (p.19)

Accomplished mathematics teachers draw on their broad knowledge of mathematics to shape their teaching and set curricular goals. They understand significant connections among mathematical ideas and the application of those ideas not only within mathematics but also to other disciplines and the world outside of school.

##### IV. Knowledge of Teaching Practice (p. 25)

Accomplished mathematics teachers rely on their extensive pedagogical knowledge to make curricular decisions, select instructional strategies, develop instructional plans, and formulate assessment plans.

### Advancing Student Learning

#### The Teaching of Mathematics

---

##### V. The Art of Teaching (p. 29)

Accomplished mathematics teachers create elegant and powerful approaches to instructional challenges. Their practice reflects a highly developed personal synthesis of their caring for students, their passion for teaching and mathematics, understanding of mathematical content, ability to apply mathematics, and rich knowledge of established and innovative educational practices.

##### VI. Learning Environment (p. 33)

Accomplished mathematics teachers create stimulating, caring, and inclusive environments. They develop communities of involved learners in which students accept responsibility for learning, take intellectual risks, develop confidence and self-esteem, work independently and collaboratively, and value mathematics.

**VII. Using Mathematics (p. 35)**

Accomplished mathematics teachers help students develop a positive disposition for mathematics and foster the development of all students' abilities to use mathematics as a way to understand the world around them. They focus instruction on developing students' mathematical power by providing opportunities for students to understand and apply mathematical concepts; investigate, explore, and discover structures and relationships; demonstrate flexibility and perseverance in solving problems; create and use mathematical models; formulate problems of their own; and justify and communicate their conclusions.

**VIII. Technology and Instructional Resources (p. 39)**

Accomplished mathematics teachers are knowledgeable about and, where available, use current technologies and other resources to promote student learning in mathematics. They select, adapt, and create engaging instructional materials and draw on human resources from the school and the community to enhance and extend students' understanding and use of mathematics.

**IX. Assessment (p. 41)**

Accomplished mathematics teachers integrate assessment into their instruction to promote the learning of all students. They design, select, and employ a range of formal and informal assessment tools to match their educational purposes. They help students develop self-assessment skills, encouraging them to reflect on their performance.

**Professional Development and Outreach**

---

**X. Reflection and Growth (p. 43)**

Accomplished mathematics teachers regularly reflect on teaching and learning. They keep abreast of changes in mathematics and in mathematical pedagogy, continually increasing their knowledge and improving their practice.

**XI. Families and Communities (p. 45)**

Accomplished mathematics teachers work to involve families in their children's education, help the community understand the role of mathematics and mathematics instruction in today's world, and, to the extent possible, involve the community in support of instruction.

**XII. Professional Community (p. 47)**

Accomplished mathematics teachers collaborate with peers and other education professionals to strengthen the school's program, promote program quality and continuity across grade levels, advance knowledge in the field of mathematics education, and improve practice within the field.

*In the pages that follow, the reader will find full explications of each standard that include discussions of the knowledge, skills, dispositions, and habits of mind that allow teachers in this field to practice at a high level.*

## Commitment to All Students

As reflected in the first of the five broad propositions of the National Board, accomplished mathematics teachers care deeply about their students and are committed to their students' learning. This commitment is apparent in every aspect of their teaching. When joined with the other standards, a rich

picture of accomplished mathematics teaching begins to take form. This vision embraces the idea that deliberate instructional decisions implemented by a caring and accomplished teacher lead to all students learning the important and challenging mathematics they need for their future.

# *Standard I: Commitment to Equity and Access*

Accomplished mathematics teachers value and acknowledge the individuality and worth of each student; they believe that all students can learn and should have access to the full mathematics curriculum; and they demonstrate these beliefs in their practice by systematically providing all students equitable and complete access to mathematics.

Accomplished mathematics teachers value all their students, treating them each with respect. They acknowledge the inherent dignity in each student and believe each has valuable contributions to make to the class. In order to provide each student with opportunities to experience challenging mathematics, they tailor their instruction to take account of students' different experiences, circumstances, language proficiencies, and needs. In doing so, teachers show that they understand how the learning of mathematics is influenced by the individually diverse backgrounds of their students.

Accomplished mathematics teachers consistently demonstrate high expectations for every student. They understand and promote the value of learning mathematics for mathematics' sake as well as the value of mathematics as a useful and powerful tool in daily life. They view mathematics as a way to open doors for students, not as a series of barriers for students to overcome. They communicate to students the power of mathematics to contribute to their intellectual growth and development, create opportunities, and provide them with a promising future. Such teachers not only declare that

all students can learn but act on this belief every day. Their most visible avenue for expressing the belief that all students can learn challenging mathematics is providing all students with the opportunity and skills to study the full range of mathematics.

In order to do this, accomplished mathematics teachers find effective ways for all students to access the mathematics being studied. They use a variety of approaches, finding the unique strengths of each student and building on those strengths. Accomplished teachers recognize when to modify the mainstream curriculum and use adaptive strategies that enable each student to contribute. They seek the support they need to modify and adapt the curriculum on an ongoing basis, thus providing a quality educational experience for all students. Teachers<sup>3</sup> find ways for all students to participate in rich learning experiences. Teachers do whatever is necessary to prevent any students from falling behind in class, seeing to it that students learn the missing facts or alternate strategies to determine mathematical facts when they need them.

Accomplished mathematics teachers recognize that cultural differences will sometimes

<sup>3</sup> All references to teachers in this document, whether stated explicitly or not, refer to accomplished mathematics teachers.



The following three standards form the foundation for decisions and actions taken by accomplished mathematics teachers and as such, they are the basis for all the other standards. Only by knowing their students well can teachers make instructional decisions that further students' learning. Only by having a deep and broad understanding of

mathematics itself can a teacher organize and deliver instruction that helps students build their own broad and deep understanding of mathematics. And only by skillfully combining their knowledge of students and mathematics with knowledge about how to teach mathematics can teachers enable students to successfully learn mathematics.

### ***Standard II: Knowledge of Students***

Accomplished mathematics teachers recognize that students are shaped by a variety of educational, social, and cultural backgrounds and experiences that influence learning. They draw on their knowledge of how students learn and develop in order to understand their students and to guide curricular and instructional decisions.

Accomplished mathematics teachers are aware of the differing ways in which students process information on their path to understanding mathematics. As they help each student construct a personal understanding of the language and processes of mathematics these teachers know that students will approach a task in many ways. Many students benefit from using concrete materials in the early stages of developing mathematical understanding, and other students may learn better from different instructional approaches throughout their mathematical experience. In designing lessons, accomplished mathematics teachers consider how students with differing aptitudes, interests, and ways of learning come to understand mathematics in varying ways, and how these students develop the reasoning processes and attitudes that characterize mathematical thinking. Teachers' instruction succeeds, in part, because of their ability to assess and anticipate students' difficulties, understanding, and misconceptions, and yet build on students' strengths. They view instruction and the learning process from a student's perspective and are thereby able to understand a student's misconception, identify the underlying rationale, and clarify the student's thinking. The

practice of accomplished teachers is distinguished by the capacity to merge the goals of the classroom and the curriculum with students' knowledge. This ability includes teachers recognizing that the numerical world view of some students differs from that of others, depending on background and culture. For example, in the United States fractions are generally taught as part of a whole, whereas in other countries they are taught as ratios. Teachers recognize these differences in starting points and accommodate them in their teaching.

Accomplished mathematics teachers recognize the variability of student development at the middle childhood through early adolescence levels. They understand the process of cognitive development in young people and know that students' cognitive profiles vary in such areas as oral and written language, logical thinking, understanding of the language of mathematics, number systems, classification systems, conceptualization, mathematical operations, mental estimation, computation, and problem solving. For example, although they might expect a young adolescent to solve complex, multistep problems, they know that more basic one- or two-step problems might be more developmentally appropriate for younger



children at the middle childhood level. They know, for instance, that middle childhood students often have a limited mathematics vocabulary to explain how a set of manipulatives is sorted and classified, whereas other students can be expected to use a richer vocabulary and provide more sophisticated explanations when given comparable tasks. As a result, this knowledge is reflected in the practice of an accomplished teacher in such areas as the complexity of problems chosen and the criteria used in assessment.

These teachers know that students who are mathematically more mature may tend to be preoccupied with new social relationships being formed with both peers and adults. Accomplished teachers know that students approaching adolescence experience many intense emotional, physical, social, and intellectual changes over a relatively short period of time and that these changes often collide with one another and sometimes interfere with students' effectiveness in the classroom. At the same time, teachers know that the energy of preadolescence may contribute to rich learning opportunities if channeled into appropriate activities. Accomplished teachers realize that not all students go through these changes at the same time, so that their instructional decisions are based on a broad knowledge of this developmental continuum rather than on generalizations about any age group. Teachers factor this developmental knowledge into their instructional planning, blending it with knowledge about how students see mathematics and develop new mathematical understanding.

Accomplished mathematics teachers identify the strengths, interests, and experiences their particular students bring to the mathematics classroom. They build on these strengths and experiences, including students' intuitive understanding of mathematical ideas, to challenge each student to achieve high levels of mathematical understanding. When students do not have the desirable prerequisite skills or have not had experiences desirable for studying a certain

concept or skill, accomplished mathematics teachers adapt their teaching to acknowledge those skills and experiences students have had. For example, when dealing with a situation that calls for using fractions with students who are weak in this area but strong in decimals, the teacher might provide a decimal-based situation more consistent with students' experiences. When working on complex problems with students who are not experienced or proficient in computation, the teacher might choose to encourage students to use calculators to allow them to learn the higher-level mathematics, while continuing to look for opportunities to build computational proficiency. When presenting a problem involving ratio and proportion, the teacher might choose to present students with a choice of problem settings, one of which he or she makes certain will match the students' experiences. And teachers know how to build upon students' strengths as they develop, using them to deepen students' understanding in mathematics and to encourage them to apply mathematical understanding to other fields.

In order to gauge their students' strengths, needs and interests, accomplished mathematics teachers insightfully observe and listen to their students in whatever setting students use to express themselves, whether in a formal classroom setting, an individual conference, or an informal conversation. These insights, together with their ability to identify students with exceptional needs or talents, enable teachers to adapt their practice equitably to meet the needs of each of their students. Accomplished mathematics teachers work collaboratively with specialists when they are available and needed. They modify their plans and materials to support each of their students, including those whose primary language is not the language of instruction. They know that, depending on their students' world view, their understanding may be more or less sophisticated than that of other students. Such teachers know their students as individuals, so they neither stereotype nor





## ***Standard III: Knowledge of Mathematics***

Accomplished mathematics teachers understand significant connections among mathematical ideas and the application of those ideas in mathematics, in other disciplines, and in the world outside of school. They have a broad knowledge of mathematical concepts, principles, techniques, and reasoning that they use to set curricular goals and shape their teaching.

Accomplished mathematics teachers have a deep and broad understanding of mathematics well beyond the level of mathematics they teach. Their knowledge emphasizes not the small details, rules, and procedures of mathematics but rather the larger themes and connecting ideas that tie together the various strands of mathematics. This rich, conceptual knowledge of mathematics allows accomplished teachers to make decisions about what mathematics is important to teach. Their knowledge base makes them well aware of where their students are headed—individually and as a group—and how to move them to continually deepening levels of mathematical understanding. This knowledge of mathematical principles, ideas, and reasoning allows teachers to monitor and adjust their teaching continuously, directing students toward important understandings that arise naturally from students' work by asking questions and guiding discourse toward these understandings. Although one of the oldest disciplines of human knowledge and thought, mathematics continues to grow and evolve. New concepts, principles, and methods become a part of the discipline each year. For example, fractals, linear programming, technology-based numerical methods and data handling, and much of the mathematics underlying cryptography have all been developed during the lifetime of many of today's teachers. Although accomplished mathematics teachers may not know all the new mathematics evolving day to day (such as fractals, linear programming, or technology-based numerical methods), they are aware of

these emerging topics. Because they understand that new mathematics outside of school has an impact on the mathematics they teach, they know the general nature of such new topics.

One of the strongest forces in the contemporary growth and evolution of mathematics is the information-processing power of modern technologies. Developments such as advanced technology in calculators and computers make simple and complex procedural tasks routine and provide effective tools for representing and reasoning about mathematical patterns. As a consequence, the infusion of technology in education is driving a steady reshaping of school mathematics curricula and teaching. Accomplished mathematics teachers are knowledgeable about the shifts in importance of numerical skills and procedures that result from powerful and accessible hand-held technological tools. They recognize that some procedural skills may need to be less well honed than in the past, and they know that a deep conceptual understanding of numbers and operations is more important than ever before for decision making and for judging the reasonableness of results when using technological tools. At the same time, they know that a ready knowledge base of numerical facts, skills, procedures and relationships can be a useful tool in dealing with many types of mathematical problems, while also understanding how technology can provide access to engaging and important problems even for students who may not have complete mastery of computational facts and procedures.



Accomplished teachers of middle childhood through early adolescence mathematics have an extensive knowledge of the discipline from several perspectives. They understand the foundations of the number system and the importance of using patterns to make generalizations and develop mathematical understanding. They know the central concepts and principles of important mathematical domains, including algebra and functions, geometry, and statistics and data analysis, and understand the conceptual foundations of discrete mathematics and calculus. They know the fundamental processes of mathematical thinking—exploration, representation, modeling, conjecture, inference, interpretation, and analysis. They understand the importance of proof and formal reasoning processes and how they work in establishing the truth of mathematical statements and in providing a standard of rigor that sets mathematics apart from other disciplines. At the same time, they understand how the development of broad reasoning processes begins with informal ways of demonstrating why particular conclusions follow from certain circumstances.

Accomplished mathematics teachers possess a deep knowledge of basic mathematical ideas. They know the power of properties of the real number system, not just as rules, but as ways for students to gain insights into operations, facts, and procedures. They see that measurement is more than simply using rulers and other tools, involving also the simple yet powerful notion of comparing an attribute of an object to a unit. They know that the transition from numerical thinking to algebraic thinking involves both an understanding of the concept of generalization and an extension of work with proportional relationships among numbers to similar relationships among quantities. And they use their understanding of statistical concepts to structure mathematically productive tasks involving data organization and analysis.

Accomplished mathematics teachers see the power of patterns in mathematics. They

recognize and generate patterns to demonstrate a variety of relationships and can identify and justify patterns they observe in complex situations. They relate patterns in one strand of mathematics to related patterns across the discipline, and they solve a variety of problems in which patterns provide insights or solutions.

Accomplished mathematics teachers know that mathematics has productive connections with other fields of human endeavor—connections that have given mathematics a remarkable history of intellectual service to problem solving and decision making across time and cultures. Today, more than ever, these connections give mathematics an important place in the interdisciplinary instruction becoming so insignificant at this level of instruction. Consequently, teachers at this level have an increased need to balance a solid knowledge of mathematics with knowledge of other content areas and the application of mathematics to those content areas. Accomplished mathematics teachers know that mathematics can be applied in a variety of settings, including other school subjects. This broad-based knowledge allows them to ensure that interdisciplinary instruction includes mathematics that not only supports those other subjects but maintains the integrity of mathematical ideas and processes that are important for students of these ages. Accomplished teachers enliven and enrich the mathematics they teach by drawing out the connections between mathematics and other aspects of students' lives and experiences.

Additionally, accomplished mathematics teachers at the middle childhood through early adolescence level understand unifying ideas such as proportional reasoning, representing mathematical relationships in multiple forms, and making generalizations from patterns. They can describe how mathematical concepts related to these ideas build from early low-level experiences through developmentally sequenced experiences toward rich, meaningful, mathematical



power. The power of mathematics lies within these big ideas as ways to describe algorithms and relationships applicable across many strands or within a strand for many situations. Accomplished mathematics teachers use their understanding of these unifying ideas and their knowledge of content across the various strands of mathematics to solve problems, describe important mathematical concepts, and apply what they know to practical situations. Proportionality, in particular, is critically important at this level, and accomplished mathematics teachers have a deep conceptual understanding of proportionality as it relates to number and operations, algebra and functions, geometry, discrete mathematics and statistics, as well as its specialization outside of school.

This broad set of understandings is an essential foundation for setting curricular goals and devising instructional activities. An accomplished mathematics teacher utilizes these understandings to help students discover concepts and principles underlying important mathematical topics, detect important relationships connecting content strands, and use mathematical ideas and methods in significant applications.

### Core Mathematical Knowledge

Accomplished teachers have extensive and connected understandings of the major ideas in the core domains of mathematics. Although their expertise may vary in degree for particular domains, these teachers have a fundamental knowledge base on which to build students' understanding of content and connections between and among mathematical strands. This knowledge includes:

#### Number and Operation Sense

Accomplished teachers understand and can model in a variety of ways basic concepts of number and operation. They understand the conceptual basis for the real number system, including the prop-

erties that unite and separate various number systems. They can describe the additive and multiplicative nature of numbers, and they know how to facilitate the transition from additive to multiplicative models. They recognize the pervasiveness of proportionality across mathematical strands and can use that concept as a model in describing a variety of situations, including those calling for ratios and percent. They understand various ways of making estimates, and they know what kinds of situations call for estimates rather than exact answers. They know how to use technology, hands-on activities, and manipulatives to support and facilitate appropriate development of numerical skills, and they can solve a variety of problems using mental processes, pencil and paper, and calculators.

#### Algebra and Functions

Accomplished teachers recognize algebra as a language for modeling problem situations and representing numerical patterns and quantitative relationships in symbolic and graphical forms. They know the importance of the link between early work with basic patterns and the later conceptual development of important ideas related to functions. They understand algebraic techniques and procedures for transforming and simplifying algebraic representations, as well as how to reason about relations and how to draw inferences in solving problems. They recognize that there are different kinds of functional relations—including polynomial, exponential, rational and trigonometric functions—and they are familiar with examples of these different kinds of relations.

#### Geometry and Measurement

Accomplished teachers understand that geometry provides a repertoire of techniques for describing, representing, and reasoning about the shape, size, measure,



and position of objects and visual patterns. They understand the characteristics of different shapes and how shapes can be related. They use geometric concepts to record and analyze properties of shapes and patterns and to study the ways those objects and patterns change when acted upon by transformations. On the basis of their knowledge of geometric relations in Euclidean and other geometric spaces, they solve problems in fields from architecture and engineering to space science and the design of communication networks. Teachers understand the structure and use of systems of measurement. They describe critical features of the measurement process and solve a variety of problems involving geometry and measurement. In the same way that they use algebraic methods to help in reasoning about geometric situations, they use visual models and methods to provide invaluable insight in thinking about patterns in quantitative and symbolic data.

### **Statistics and Data Analysis**

In our information-rich and technologically advanced society, it is of vital importance to their social awareness and career prospects that students be able to collect, represent, and interpret data for such purposes as testing hypotheses, drawing inferences, and solving problems. The study of statistics should solidify, deepen, and build on students' understandings of exploratory data analysis.

Accomplished teachers use a variety of conceptual and procedural tools for collecting, organizing, and reasoning about data. They know a variety of formats for collecting and reporting data and they understand basic principles of inference. They apply these tools, including numerical and graphical techniques for representing and summarizing collections of data, to interpret and draw inferences from these data, and to make decisions in a wide range of applied prob-

lem situations. They do more than use statistics as a way to describe data; they use statistical methods to make generalizations about samples based on the methods and language of probability.

### **Concepts of Discrete Mathematics**

Whereas calculus and other continuous mathematics strands such as algebra, geometry, and trigonometry are used to model the physical world, the processing of information requires the use of discontinuous, or discrete mathematics, the study of sets and systems containing a finite number of elements.

Accomplished mathematics teachers apply algorithmic thinking to solve problems involving discrete data. They represent problems using matrices, finite graphs, and tree diagrams. They use counting techniques to enumerate possibilities involving order and combinations. They can describe basic algorithms for doing everyday tasks and use technology to solve a variety of discrete mathematics problems in practical settings.

### **Concepts of Calculus**

Accomplished teachers know that calculus provides a language for modeling dynamic change. They have a basic understanding of conceptual ideas related to calculus, including the ideas of limit, continuity, maximum, and minimum. They use tabular, graphical, symbolic, and applied contexts to develop the concept of optimization in practical settings, using technological tools.

### **Mathematical Thinking Processes**

Mathematics is often described by naming important concepts, facts, and operations in its major topic strands. But there are characteristic mathematical thinking processes that apply in all of the topical strands.



Accomplished teachers of mathematics understand and are able to demonstrate the use of mathematical processes such as—

- the use of strategies and concepts for discovering and describing patterns in visual, numerical, and symbolic data, (i.e., processes such as classification, representation, and inductive reasoning and concepts such as symmetry, similarity, randomness, stability, recursion, and continuity);
- the use of methods of formal verification for mathematical conjectures, including rules of logical inference and proof strategies;
- the modeling of mathematical relations in problem situations by using symbolic expressions—representing important relationships, operating on symbolic expressions to gain understanding of the situation or to draw inferences about it, and applying results of mathematical analysis to solve problems and make decisions;
- the use of heuristics for solving mathematical problems (e.g., testing extreme cases, using guess-and-check methods, conducting an organized search of specific examples, or using visual problem representations);
- the use of calculating and computing technologies to search for patterns in numeric, graphic, and symbolic data;
- the use of strategies for communication of mathematical information in verbal, numerical, graphical, and symbolic forms and through physical models of mathematical principles.

### Contexts for Mathematics

In one sense, mathematics is the most abstract of disciplines. But many of its most interesting abstract concepts, structures, and operations have arisen from or found embodiment in patterns of objects and actions in scientific,

technical, economic, or practical situations. Accomplished teachers understand significant connections of mathematical ideas to other disciplines and practical fields and use this understanding to make wise curricular and instructional decisions. They use their understanding of the historical and cultural roots of mathematics and their awareness of non-Western concepts of mathematics to help students appreciate the development of mathematics by human beings throughout history and across cultures. Their knowledge of mathematical contexts includes not only a historical perspective but also a basic understanding of the applications of mathematics. They know the value of such applied contexts, as well as purely theoretical contexts, in motivating the study of engaging problems. They also know what mathematical modeling means with regards to the use of mathematics not only to represent a situation (as with an equation) but also to analyze the situation (as by exploring its properties, attributes, or possible solutions).

In addition to the array of fundamental thinking processes that apply across all the topic strands of the subject, mathematics is woven together by important conceptual structures that occur in many different topic strands, giving a coherence to the subject that is powerful support for teaching, learning, and applied problem solving. For example, matrices are invaluable tools for recording and reasoning about complex data sets in algebraic systems of equations, organizing and analyzing data, and making geometric transformations. Knowledge of proportionality helps teachers understand basic numerical operations, the algebraic concept of slope, similar geometric figures, modeling, and consumer topics such as discount and sales tax. The study of the concept of equivalence helps teachers understand many mathematical ideas, including properties of numbers and operations, solving algebraic equations, and dealing with congruent figures. To make the power of these structural connections available to



# Standard IV: Knowledge of Teaching Practice

Accomplished mathematics teachers rely on their extensive pedagogical knowledge to make curricular decisions, select appropriate instructional strategies, develop instructional plans, and formulate assessment plans.

Accomplished mathematics teachers use their strong base of pedagogical knowledge for making curriculum decisions, designing instructional strategies and assessment plans, and choosing materials and resources for mathematics instruction. This knowledge base is constantly expanding and evolving in order to include new development in mathematical pedagogy, curriculum, and assessment and reflects the teacher’s ongoing professional growth. On the basis of what accomplished teachers know about effective teaching practice, they continually refine how they think, act, and make decisions.

Accomplished mathematics teachers are knowledgeable about techniques for working with students in groups and about selecting tasks appropriate for group work. For students of different backgrounds who may not be used to working together, these teachers find ways to facilitate their comfort and participation in group process. They know when to structure group tasks tightly and when to allow less defined activities. They know how to form groups in which students have an opportunity to learn for themselves and from each other. They acknowledge that individual work is important while recognizing the importance of learning how to work together on mathematical problems. A teacher working with students in small groups might have to spend a good deal of time helping students learn to work together, involve other group members, and organize for a particular group task. On the other hand, a middle school teacher with students who have been working in groups for several years might focus more on the level of mathematical generalization that students offer.

However, if these older students lack experience working in groups, they might need a structure similar to that required by younger students. Conversely, some students might need help learning how to work alone or to trust their own intuition rather than rely on direct instruction. The accomplished mathematics teacher takes into account the individual needs and developmental levels of students when designing instruction and knows how to pull students’ ideas together and build on them.

Accomplished teachers apply their knowledge of teaching practice to the teaching of mathematical concepts and skills, and they select approaches appropriate to their students and the given situation. They know how to integrate various models of assessment as part of their teaching, and they focus predominantly on meaningful, in-depth tasks rather than emphasize low-level, rote, mechanical skills.

Accomplished mathematics teachers know that at the middle childhood through early adolescence level, the various subject areas can enhance each other. Consequently, in selecting tasks and planning instruction, they are adept at connecting mathematics to science, social studies, the arts, physical education, language arts, and other fields. Whether teaching in a self-contained classroom, participating in an interdisciplinary team, or teaching in a mathematics classroom, these teachers use their knowledge of mathematics and the larger curriculum to connect various threads of learning through rich instructional tasks that cross disciplines.

In teaching mathematics, accomplished mathematics teachers stress the importance



of fostering conceptual development and deep understanding. They maintain a delicate balance between skills practice and this deeper understanding, frequently embedding such skills practice into rich problem-solving opportunities that in turn reflect the world of their own students. They focus on motivating students on the basis of students' needs, interests, and intrinsic motivation, rather than imposing teacher-selected models. For example, whereas some students may respond positively to competition or timed practice, others may get frustrated or even develop long-lasting negative attitudes about mathematics when using such techniques. And teachers know that some students have a cultural aversion to feeling conspicuous. Accordingly, accomplished mathematics teachers choose alternative methods to motivate these students.

Accomplished teachers rely on a large repertoire of problems, demonstrations, experiments, questioning strategies, manipulatives, and instructional resources to provoke learning. They factor into their instruction the understanding that students' prior knowledge and experiences, along with their cognitive development, contribute to their readiness for new challenges. For example, a middle childhood teacher might develop the concept of the area of a rectangle by asking students to arrange twelve plastic tiles in as many ways as possible to form a

rectangle with an area of twelve square units, draw pictures on grid paper of the resulting configurations, and discuss their observations. On the other hand, teachers of more sophisticated students might approach a similar lesson in a more abstract way by facilitating a whole-class discussion during which the class creates a chart of all the possible whole-number combinations for the length and width of a rectangle with an area of twelve square units and explores the relationship between the length and the width. Such teachers also anticipate where students are likely to have difficulties, especially when a lesson runs counter to students' experiences or world view; know when it is important to let students struggle with a problem and when to offer assistance; recognize the value of students' confronting conflicting ideas; and draw on their repertoire to provide questions and tasks that will build student understanding. In short, these teachers know when to intervene with an activity and do so in ways that make students feel safe to ask questions and discuss their ideas.







## The Teaching of Mathematics

How a teacher makes decisions and implements plans in a flexible, appropriate manner in the classroom provides the most visible and, arguably, the most important demonstration of excellence in teaching. The next five standards create a picture of mathe-

tics teaching, including the environment the teacher creates for learning, the nature of the mathematics taught, the resources used to teach, the power of technology as a resource, and the practices teachers use to assess and monitor learning.

# ***Standard V: The Art of Teaching***

Accomplished mathematics teachers create elegant and powerful approaches to instructional challenges. Their practice reflects a highly developed personal synthesis of their caring for students, passion for teaching and mathematics, understanding of mathematical content, ability to apply mathematics, and rich knowledge of established and innovative educational practices.

Accomplished mathematics teachers are Artists, with a love of teaching and a commitment to its practice that is evident in all they do. Not all such teachers practice their art in the same way, but all of them pull together their knowledge of students, mathematics, and teaching into a cohesive model of accomplished and exciting mathematics teaching.

Accomplished mathematics teachers not only place a high value on mathematics, they take joy in it, are excited by the ideas they explore with students, and communicate their excitement to their students. They design intellectually challenging lessons that enable their students to learn important mathematics and to whatever extent possible, they involve students in the learning process. They carefully guide, direct, and encourage decision making, mathematical reasoning, and insight through questioning, discussion, and written communication. When necessary, they teach students study skills designed to enhance their personal approach to an understanding of mathematics. They listen to students and are prepared to adapt instruction to accommodate unexpected tangents, and they know that success at a piece of a task often motivates students to attempt the next level of complexity. They orchestrate and facilitate learning and stimulate their students'

intellectual curiosity. They encourage their students to formulate and test mathematical conjectures, for example, in order to expand the students' thinking. In approaching teaching challenges, whether from students, content, or other factors, these teachers use their expertise to address problems effectively. Their solutions and decisions may be based on well-accepted principles or on creative new approaches.

Accomplished mathematics teachers constantly monitor and adjust their teaching. They capitalize on opportunities generated by unexpected but relevant student questions or discussions to pursue important, challenging mathematical ideas. They observe and listen to students in order to know when a student misunderstands, adapting their instructional plan to clarify misconceptions and facilitate mathematical learning. They modify and adjust the pacing and mode of instruction, and they know how, when, and to what extent to nurture their students to accommodate them and promote their learning.

Accomplished mathematics teachers appreciate the diversity of their students. They seize the opportunity provided by students' different experiences and cultural backgrounds to build a picture of mathematics as something that all people use to express themselves and solve



problems. They build on students' individual strengths, background, world view, and intuitive knowledge to guide them to higher and more sophisticated levels of mathematical thinking and understanding.

Accomplished mathematics teachers model a positive disposition toward mathematics, showing that they truly enjoy mathematics and take pleasure from teaching it. They ask interesting and thought-provoking questions that stimulate students' interest in mathematics, and they provide students with engaging tasks that are well matched to the students' level of mathematical literacy. They deliberately select and design tasks that focus on meaningful, challenging mathematics, sometimes centered on applications and sometimes designed to build an understanding of important mathematical ideas and relationships.

Accomplished mathematics teachers model appropriate mathematical discourse and questioning. They facilitate the development of students' mathematical communication skills, including purposeful attention to mathematical conversation. In doing so, they may assume a variety of roles, at different times serving as a facilitator of student inquiry, an information provider, or a collaborator with students. They give students frequent opportunities for multiple forms of communication and to write and speak mathematically in order to explain their thinking, make generalizations about their explorations, justify their conclusions, and describe their attitudes and feelings about mathematics. They use communication in the mathematics classroom as a way to support the development of students' broader language skills.

Accomplished teachers understand that students differ in their levels of cognitive development in various domains and that such differences will be reflected in students' performances. They are able to include students of all skill levels into their lessons by providing multiple entry points and

approaches. For example, if a middle childhood teacher engages students in a bar-graph activity to determine their preferences for flavors of ice cream, the students might create their graph by each placing an adhesive note on a prepared grid on the chalkboard or by making a human bar graph. The teacher might ask students to interpret the displays, read information directly, and make predictions based on the information available. Students who have already had these experiences, on the other hand, might be expected to collect this data by surveying their classmates and displaying the data in the form of a paper-and-pencil bar graph. These students might be asked to extrapolate and apply information from the graph and determine the impact of new data on existing data. Accomplished teachers' lessons reflect their knowledge of students' prior experience, the students' analytical skills, and the complexity of the task at hand. In the discussions included in their lessons, they are particularly skilled in asking questions, knowing what kinds of questions will guide students' explorations, what kinds of questions will facilitate students' understanding, and what kinds of questions might spark an independent investigation.

Overall, accomplished mathematics teachers create classrooms where mathematics and learning are priorities, where there is excitement about mathematics, and where students understand the conceptual underpinnings of mathematical ideas and demonstrate proficiency in important mathematical skills. As a result of the teachers' artistry in teaching, students enjoy being in these teachers' classrooms, feel confident about their own mathematical ability, and look forward to opportunities to demonstrate and extend their own knowledge.







## *Standard VI: Learning Environment*

Accomplished mathematics teachers create stimulating, caring, and inclusive environments. They develop communities of involved learners in which students accept responsibility for learning, take intellectual risks, develop confidence and self-esteem, work independently and collaboratively, and value mathematics.

**T**he learning environment in an elementary or middle school classroom consists of more than the mere physical surroundings and the arrangement of desks or tables. It also encompasses the atmosphere with regard to the subject matter and the students. The atmosphere in the classroom of an accomplished teacher is one in which students feel safe to take risks and where they enjoy and succeed in doing mathematics. In such a classroom it is clear that every student is involved in challenging mathematics, that diverse points of view are valued, and that students are expected to learn to work alone as well as together. The classroom operates as an intellectually challenging and thoughtful mathematical community where all citizens are valued and where all participate in the work and life of the community.

Accomplished mathematics teachers create classrooms where students work on engaging mathematical tasks in small groups, in large groups, and individually, with a focus on the importance of learning to work with others on mathematical problems. Students in these classrooms develop a strong sense of self-confidence as they see their competence in mathematics growing and discover that they can successfully deal with increasingly challenging problems. Over time they come to understand that it is safe to take risks when offering ideas, opinions, or solution strategies. In the classes of accomplished teachers, every student's ideas are valued, and students know this. These teachers help students feel comfortable with their levels of expertise in mathematics, while at the same time helping students recognize that they can expand their understanding.

Because of the value placed on students' ideas and because of the nature of mathematics instruction in these classrooms, students see mathematics as something invented and developed by people. Their teachers provide them with direct opportunities to construct their own mathematical meanings and even their own procedures for mathematical operations. Teachers achieve this result by giving students frequent experience in working with materials, making hypotheses, and explaining and justifying their conclusions. Students know that sometimes mathematics requires extra effort and that sometimes even mathematicians and teachers may be unsure about how to solve a problem and may make mistakes. Consequently, they are not afraid to try a new approach, knowing that it is all right if they are not initially successful. They are comfortable and confident enough to ask each other for help, offer ideas or suggestions, and work either alone or with others as they travel well-worn paths of common procedures or delve into the unknown territory of a mathematical idea that is new to them.

Because students are valued members of the learning community, accomplished mathematics teachers create a positive environment where students can work comfortably. They maintain high expectations for student behavior, demanding that it support the learning of mathematics. Nonetheless, these teachers know that sometimes discipline problems arise in the day-to-day operation of the classroom. Accomplished teachers know how to limit the number and severity of these interruptions and deal with such problems quickly and fairly so



## ***Standard VII: Using Mathematics***

Accomplished mathematics teachers help students develop a positive disposition toward mathematics and foster the development of all students' ability to use mathematics as a way to understand the world around them. They focus instruction on developing students' mathematical understanding by providing opportunities for students to investigate, explore, and discover structures and relationships, demonstrate flexibility and perseverance in solving problems, create and use mathematical models, formulate problems of their own, and justify and communicate their conclusions.

**M**athematics at all levels is a blend of concepts, ideas, procedures, reasoning processes, and attitudes. Its tools include representation, modeling, proof, experimentation, classification, discovery, visualization, application, and computation. Its practice includes mental exercise and pencil-and-paper representation, yet it has been profoundly affected and extended by technology, including both computers and calculators. Accomplished teachers understand how important general concepts and reasoning methods mediate the use of these tools, guide practice, underlie the search for patterns, and lead to the creation of new mathematics. They have a deep understanding of the broad, connecting ideas of mathematics and use this knowledge to help students build their own mathematical tool kit on a solid foundation of mathematical understanding. Though these teachers are knowledgeable about and sensitive to developmental issues in their students, they do not adhere to a fixed sequence that prescribes all aspects of instruction.

Accomplished mathematics teachers interweave mathematics concepts throughout the curriculum using a rich and varied set of tasks, often beginning with manipulative activities and connecting to symbolic ones. Such teachers skillfully use visual and kinesthetic tools, such as interlocking cubes or pattern blocks, to help students model real-world problems and mathematical ideas. They help students make the connection between their manipulative experiences and the mathematical ideas they

need to grasp. The accomplished mathematics teacher might choose to have students use wooden cubes or a number line to develop certain computation concepts, depending on the students' developmental level. As students become more sophisticated in their mathematical ability, the teacher places greater emphasis on developing their ability to do such computations mentally. For example, a class of middle childhood students might be asked to estimate the total cost of two or three grocery items, whereas other students might be asked to determine whether they have enough money to purchase a longer list of items. The accomplished teacher will notice when a middle childhood or early adolescent student is having difficulty at an abstract level, and he or she might therefore choose to use a concrete or representational approach. Such teachers integrate manipulatives and other materials into a comprehensive program in which the focus is on students' learning important mathematical concepts, procedures and skills, and in which mathematical modeling is used as a routine way to interpret situations in the world.

Accomplished mathematics teachers promote and model the use of various styles of informal reasoning and emphasize its importance as an underlying foundation across mathematical topics. They use a variety of tools to help students develop reasoning skills, including the ability to make and justify simple conjectures, explain the thinking behind solutions to problems, and formulate convincing arguments for



generalizations and conclusions. Mathematical reasoning permeates the tasks and actions in the mathematics classes of accomplished teachers. Such teachers regularly have students convince one another of the validity of particular representations, solutions, conjectures, and answers through both writing and discussion. They demonstrate their own mathematical reasoning in their classrooms, encouraging students to question and extend their logical arguments and providing a safe arena in which students can offer counterexamples to or questions about those arguments. Through such experiences, teachers promote the idea that mathematics represents a different way of thinking and that individual students' approaches to mathematical thinking can vary greatly. They also communicate the idea that mathematics provides a set of powerful reasoning and problem-solving tools, rather than espouse the historical view that mathematics is a series of procedures to be learned.

Accomplished mathematics teachers ensure that students use both written and oral language to describe and discuss their mathematical thinking and understanding, providing them frequent opportunities to listen to, respond to, and question the teacher and one another in the process of discussing mathematical ideas, developing mathematical understanding, and solving mathematical problems. The tasks these teachers construct, which are appropriate to students' development levels and available resources, include regular opportunities for students to read mathematical materials (including textbooks and computer-generated materials) and respond to questions based on their reading. Since many middle childhood and young adolescent students may not be highly accomplished readers, teachers provide such students specific guidance on how to read their mathematics books and on how to access mathematical reference materials, either in print or on-line in a computer database.

Accomplished mathematics teachers deliberately design opportunities for students to

use and develop appropriate mathematical discourse as they reason and solve problems, through the use of probing yet supportive questions and through the structure used in the classroom. They provide opportunities for students to talk with each other and work together in solving problems, recognizing that students' justifications and contributions to a group enhance their own learning. Problem solving at the middle childhood through early adolescence level of mathematics should closely relate to students' everyday lives, include opportunities for students to formulate and solve their own and each other's problems as they use mathematics to make sense of their world, and make them aware of their progress individually and in group situations. In addition, accomplished teachers provide some problems that are strictly mathematical in nature, such as problems dealing with the exploration of numerical patterns within the multiplication table. Accomplished mathematics teachers realize that significant mathematical learning takes place when it is built on students' understandings; consequently, they structure tasks and ask questions that deliberately elicit student strategies and thinking.

Accomplished mathematics teachers encourage investigation, cooperation, and communication in order to promote both problem solving and problem posing. By doing so they help their students develop a sense of reflection about their work in which they make good use of available resources, reasonable choices of methods for working on problems, and sound decisions about which approaches to pursue or abandon. Accomplished teachers also find ways to embed skills such as computation in meaningful problem-solving contexts. Students in the classes develop a growing repertoire of problem-solving strategies and procedural facility, using appropriate tools, that serves them broadly both in and outside of mathematics classes. Students in the classes of accomplished teachers do not abandon a problem if they do not immediately know an





# *Standard VIII: Technology and Instructional Resources*

Accomplished mathematics teachers are knowledgeable about and, whenever possible, use current technologies and other resources to promote student learning in mathematics. They select, adapt, and create engaging instructional materials and draw on human resources from the school and the community to enhance and extend students' understanding and use of mathematics.

Accomplished mathematics teachers utilize a variety of instructional resources to accomplish their goals. They know where to find commercially available resources for a specific purpose and how to adapt when those materials are not available, either by constructing their own materials or selecting alternative materials. They make wide use of the human resources in their community to enhance their instruction, bringing in speakers, asking the community to share print or media materials that might support mathematical goals, and otherwise utilizing people in the school and community in a variety of constructive ways.

Accomplished mathematics teachers know the importance of developing mathematical concepts concretely, so they are knowledgeable about the use of a variety of manipulative materials that support their instructional goals. Although not every teacher will use the same materials for the same purposes, accomplished mathematics teachers are adept at using concrete materials that help students develop various mathematical understandings.

Accomplished mathematics teachers are skilled at selecting appropriate print materials, both textbook and supplementary. They know how to evaluate materials to determine what best supports their instructional goals. Such teachers do not feel limited to content or approaches in their textbooks; they are comfortable using other resource materials as a regular part of their teaching.

Accomplished mathematics teachers know about and are proficient in the use of current technologies, including recent advances in computers and calculators, and understand the

use of other emerging technologies. Recognizing that the very content and practice of mathematics is affected by calculators and computers, they are advocates for the ongoing technological improvement of their classrooms and schools. They use their knowledge of technology and their understanding of mathematics to make decisions about what kinds of technology to use, what kinds of instructional materials are most appropriate with various tools, and what mathematics is important given the expanding capabilities of technology.

Accomplished mathematics teachers look for powerful applications for computers, calculators, and other technologies rather than simply replace low-level worksheets with low-level computer exercises. When possible, they use basic computer tools such as spreadsheets, databases, and graphing software to allow students to expand their mathematical tool kit. They encourage students to use telecommunications to access information and communicate with other students. They may select software that simulates real situations in order to address problems that might be otherwise inaccessible, or they may use high-level mathematical problem-solving software to push students toward new insights.

In dealing with calculator use in elementary and middle school classrooms, accomplished mathematics teachers know the importance of students' learning the meaning of the basic operations as well as developing strong number sense. Only with these understandings can students use technological tools in appropriate ways. Teachers understand, however, that withholding such tools from students until they



## Standard IX: Assessment

Accomplished mathematics teachers integrate assessment into their instruction to promote the learning of all students. They design, select, and employ a range of formal and informal assessment tools to match their educational purposes. They help students develop self-assessment skills, encouraging them to reflect on their performance.

Accomplished mathematics teachers view ongoing assessment as an integral part of their instruction, benefiting both the teacher and the student. They skillfully incorporate opportunities for assessing students' progress into daily instruction, observing students as they produce a project or complete a task, interviewing a group of students as a final step in an assignment, or watching students present their work to a panel of citizens or parents. They use a variety of strategies to probe and push their students' thinking, especially by asking questions designed to encourage students to extend their understanding of mathematical ideas.

Accomplished mathematics teachers are knowledgeable about a variety of approaches to assessment, including testing, performance assessment, observation and analysis of student behavior, interview techniques, and other evolving strategies for monitoring how students are growing in their mathematical understanding. They select specific types of assessment on the basis of their students' needs, the mathematics being studied, and the instructional approach used, among other considerations. They may administer classroom assessment measures to groups of students, but they also rely on individual assessment to monitor student progress. They recognize that the form of assessment is not nearly as important as how well it relates to classroom instruction, and they use the results of assessments to inform their instructional decisions and improve their teaching. In time, the students of accomplished teachers come to view assessment as an important means of evaluating their own progress and may not even distinguish it from instruction.

Equally important to the selection of appropriate assessment methods is that accomplished mathematics teachers provide students with regular opportunities to reflect on what they have learned and are learning. They know that students must become self-reliant learners and thus help them develop in that direction beginning in elementary school. Students know what to expect when they are assessed and are often given opportunities for open-ended reflection about a task, a unit, an experience, or their learning in general. Both middle school students and elementary students can begin to take responsibility for their own learning, especially if they are given such opportunities in their classes. These opportunities for self-reflection might come in the form of entries in a journal, discussions with other students, or notes to the teacher. For example, at the end of a lesson students might be asked to write a brief summary of what they learned. Accomplished mathematics teachers understand that the richness of student responses will vary according to each child's level of cognitive development in such areas as written language, understanding of the language of mathematics, and conceptualization.

Accomplished mathematics teachers may have to deal with situations in which a school, district, or state insists on using tests or instruments that do not support important mathematics learning goals. In such external testing situations, accomplished mathematics teachers do not abandon their instructional goals and allow such testing to disrupt their instruction; rather, they incorporate the pertinent mathematics objectives of the test into their overall instructional plan. At the same time, they advocate changes in accountability measures so that



## Professional Development and Outreach

As described in the five broad propositions of *What Teachers Should Know and Be Able To Do*, accomplished mathematics teachers think systematically about their practice and learn from their experiences. As professionals, they reflect on their own teaching, periodically altering course to respond to new ideas and the rate of progress they observe in their students. But excellence and growth in the classroom are not enough in today's world. Persistence and creativity in involving families who may be reluctant to participate in school are critical in supporting students' learning. And professional teachers today—more than at any other time in history—are being asked to

reach beyond their classroom to participate in broader learning communities and out to the community at large. Whether at the school, district, state, or national level, the involvement of accomplished mathematics teachers is vital in improving educational opportunities and learning for all students. The definition of who and what a teacher is continues to evolve toward a picture of a talented and competent professional who is charged with the most important function in society—educating tomorrow's adults. Important signs of this evolving professionalism are reflected in this next cluster of standards.

# Standard X: Reflection and Growth

Accomplished mathematics teachers regularly reflect on teaching and learning. They keep abreast of changes in mathematics and mathematical pedagogy, continually increasing their knowledge and improving their practice.

For accomplished mathematics teachers, every experience with a group of students provides an opportunity for reflection and growth. When things go well, they think about how to apply the lessons learned to other situations; when things do not, they consider how to improve. Such would be the case even if the world of mathematics were static.

However, as discussed earlier, the body of mathematics itself and the pedagogical bases for teaching mathematics are changing rapidly. New mathematical fields are opening up with increasing frequency, and they ultimately have effects at the school level: Witness the ways in which courses in discrete mathematics, exploratory data analysis, and problem solving have reshaped the curriculum within the past decade. Likewise, current professional wisdom suggests instructional practices that are substantially different from those that have predominated during the past half-century.<sup>4</sup> Indeed, we can expect an evolution of teaching practices as our knowledge of thinking, teaching, and learning grows. Thus, those teachers who stand still are

in fact moving rapidly backward, and even the most accomplished mathematics teachers must continue to refine their practice.

Accomplished mathematics teachers devise and employ a variety of strategies to regularly gather information about their teaching; for instance videotapes of lessons, observations of students' responses to particular topics and teaching methods, conversations with students, test scores, attitude surveys, peer observations with colleagues, or their own quiet reflection over the events of the day. They critically examine the value of their plans in light of this information and modify their teaching practices on the basis of their experiences and this continuous process of self-examination. They seek out information that can be useful to the ongoing improvement of their practice; this includes observing the accomplished practice of colleagues.

Accomplished mathematics teachers keep abreast of changes in mathematics and mathematical pedagogy, actively working with colleagues to expand their knowledge and

<sup>4</sup> See, for example, National Council of Teachers of Mathematics, *Professional Standards for Teaching Mathematics*, Reston, VA: 1991.



## ***Standard XI: Families and Communities***

Accomplished mathematics teachers work to involve families in their children's education, help the community understand the role of mathematics and mathematics instruction in today's world, and, to the extent possible, involve the community in support of instruction.

Recognizing the complexity of the family-school-student relationship, accomplished mathematics teachers view families as partners who can encourage children to persevere in mathematics. They value and respect the role of families as students' first teachers and as supporters of students' growth and development. Teachers also look to families for information about students' strengths, interests, dispositions, habits, and home life. They actively seek to learn about the various cultures from which their students and their families come, respecting their values and recognizing that cultural differences may have an impact on instruction. When families hold values the teacher sees as counterproductive to learning, he or she works closely with the families to understand the differences and create a supportive environment conducive to student learning. For example, teachers may have to spend a good deal of time working with families who think students should be studying mathematics or methods he or she does not consider appropriate. In these cases and in others in which teachers and families have differing opinions, accomplished teachers work to find common ground and to achieve mutual understanding and respect that support the best interests of students.

Accomplished mathematics teachers help families by providing them with information they need to understand school programs, the significance of test scores and grades, the importance of regular attendance, the reasons for group or class assignments, and the benefits of planning for future education, including the consequences of taking or not taking certain courses. Accomplished teachers often

design homework assignments and provide other resources and opportunities that involve activities to encourage family discussion of school subjects, being mindful that not all students have families to assist them. These teachers work with families to help their children develop good learning habits and study skills, complete homework, set goals, and improve performance.

Accomplished mathematics teachers provide support and encouragement when students are doing well and mobilize efforts to help students who are having difficulty. They advise parents or guardians of student progress on a regular basis and respond to family concerns. They help families understand and appreciate the goals and objectives of mathematics instruction, both in the short and long term, and make them aware of useful and available materials and programs that will broaden students' mathematical experiences.

Accomplished mathematics teachers realize that the best support for good mathematics instruction often comes from significant adults in students' lives who are informed about mathematics education. Activities to inform and involve adults might include creating a welcoming environment that opens the school to families; inviting family members to participate in their children's classrooms; working with parent advisory groups; conducting special school activities that involve students and families working together to do mathematics; supporting mathematics fairs and other events that allow students to demonstrate their mathematical skills to the larger community; and involving families in career days.



## *Standard XII: Professional Community*

Accomplished mathematics teachers collaborate with peers and other education professionals to strengthen the school's program, promote program quality and continuity across grade levels, advance knowledge in the field of mathematics education, and improve practice within the field.

Accomplished mathematics teachers do not work in isolation. They have much to contribute to the quality of practice of their peers and to the design of the overall instructional program of the school. Moreover, they gain much by collaborating with families, colleagues, administrators, and other members of the community in the education of students.

The commitment of teachers to a school begins with dedication to students and their learning of mathematics. Consequently, the focus of a teacher's contribution to the larger learning community often revolves around curricular issues. By periodically reviewing district and school curricula, accomplished mathematics teachers can help ensure that they are thoughtfully organized, consistent with the profession's views of the best practices, and tailored to the students and community they serve. Accomplished mathematics teachers can also work effectively with other school professionals to make students feel part of a community with a coherent ethos and to help them integrate knowledge across the disciplines. They may work with language arts teachers to share ideas on communication skills and writing in mathematics; with social studies teachers on incorporating problem solving or data analysis; with visual arts teachers on problem identification, spatial relationships, and graphic representation; and with science teachers on investigations and analyses. They may function as part of an interdisciplinary team or deal with all disciplines in a self-contained classroom.

Accomplished mathematics teachers see themselves as partners with other teachers dedicated to improving the profession. They care about the quality of practice of their

colleagues and take a special interest in assisting both novice and experienced teachers as they mature into accomplished practitioners. Their commitment to accomplished teaching in their schools and communities is continuous and explicit. They observe and study other teachers' practices, engage colleagues in dialogue about professional issues, and may serve as mentors to new teachers as well as coaches to experienced colleagues. They work with administrators both to inform them and to enlist their support for improving mathematics instruction.

As an extension of their local responsibilities, many accomplished mathematics teachers engage in a wide range of other professional activities. They may make contributions to the advancement of their field through participation in local, regional, state, and national organizations. They may work to support the improvement of mathematics education at the postsecondary level and to support the professional growth of teachers of mathematics by conducting workshops, writing about practice, or undertaking action research. Such commitment is central to accomplished teachers' dedication to the quality of their own practice and the advancement of mathematics education.





Not all mathematics teachers practice in ideal teaching environments. Not all mathematics teachers have well-equipped classrooms, reasonable class sizes, or even physically conducive surroundings. Nonetheless, accomplished mathematics teachers practice successfully in all kinds of situations.

The twelve standards in this report provide a profile of the accomplished Middle Childhood through Early Adolescence/Mathematics teacher. Although in sum they are challenging, every day these standards are upheld by teachers like the ones described in these pages, who inspire and instruct the nation's youth. Too many go unnoticed and unappreciated, a situation National Board Certification holds the potential to change.

These standards also promise to be a stimulus for self-reflection on the part of teachers at all levels of performance and a catalyst for stimulating a healthy debate and the forging of a new professional consensus on accomplished practice in this field. If these standards can advance the conversation about accomplished teaching, they will provide an important step toward the National Board's goal of improving student learning in America's schools.



# Middle Childhood through Early Adolescence/ Mathematics Standards Committee



**Leslie G. Mancuso**—Chair  
K–12 Mathematics Coordinator  
McAlister Middle School  
Suffield, Connecticut

**Gladys N. Sanders**—Vice Chair  
K–12 Mathematics Coordinator  
Lawrence Public Schools  
Lawrence, Kansas

**Joan E. Carlson**  
Mathematics Teacher  
Mendocino Middle School  
Mendocino, California

**Cindy Chapman**  
Second- and Third-Grade Teacher  
Inez Elementary School  
Albuquerque, New Mexico

**Joseph R. Georgeson**  
Mathematics Teacher  
Glen Hills Middle School  
Glendale, Wisconsin

**Jerilyn R. Grignon**  
Assistant to the President  
College of Menominee  
Keshena, Wisconsin

**Jeremy Kilpatrick**  
Professor  
Department of Mathematics Education  
University of Georgia  
Athens, Georgia

**Katherine K. Merseth**  
Executive Director  
Harvard Project on Schooling and Children  
Harvard University  
Cambridge, Massachusetts

**Perry Montoya**  
Sixth-Grade Teacher  
Frost Elementary School  
Chandler, Arizona

**Sallie Morse**  
Mathematics Teacher  
Union Mills School  
Union Mills, North Carolina

**Lori Murakami**  
Fourth- and Fifth-Grade Teacher  
Clarendon Alternative School  
San Francisco, California

**Cynthia H. Petti**  
Sixth- and Seventh-Grade  
Mathematics Teacher  
Gifted and Talented Coordinator  
Great Salt Bay Community School  
Damariscotta, Maine

**Christine D. Rosen** (Retired)  
Second- and Third-Grade Teacher  
Langley-Park McCormick School  
Hyattsville, Maryland

**Edward A. Silver**  
Senior Scientist  
Learning Research and  
Development Center  
University of Pittsburgh  
Pittsburgh, Pennsylvania

**Uri Treisman**  
Professor  
Department of Mathematics  
University of Texas at Austin  
Austin, Texas



The development of these standards for National Board Certification of Middle Childhood and Early Adolescence/Mathematics teachers represents a singular achievement, as it marks the first time educators from the variety of perspectives that compose the professional community have come together to forge a consensus about the critical aspects of accomplished practice in this field. The standards are the product of the Middle Childhood and Early Adolescence/Mathematics Standards Committee and the many people who worked with the committee to craft them, including the NBPTS Board of Directors and its Certification Standards Working Group (later the Standards and Professional Development Working Group), which provided the process with careful guidance, helpful criticism, and much encouragement. The result is that the Middle Childhood and Early Adolescence/Mathematics Standards Committee has created advanced teaching standards that will speak clearly and powerfully to prospective candidates for National Board Certification, to the mathematics community, to the larger education community, and to the public. NBPTS is most appreciative of the committee's work.

Many individuals and institutions contributed to this effort. Teachers and other scholars, administrators, state and local officials, education association leaders, and others from across the country critically reviewed draft standards, provided sound advice about how to strengthen the standards, and helped position the standards so they might complement other initiatives designed to advance practice in this field. Whereas a variety of people unstintingly provided the National Board with caring, intelligent, and imaginative counsel, several deserve special thanks. The members of the Middle Childhood and Early Adolescence/Mathematics Standards Committee, a group of exceptionally dedicated, creative, and industrious professionals, explored their rich experience to yield this new vision of accomplished practice. They were ably led by Leslie Mancuso and Gladys Sanders, chair and vice chair, respectively, who kept the teacher's perspective in the forefront and helped forge a consensus around large principles when lesser solutions were also available. Kathy Layton, Judith Rohde, and Leo Ramirez, the Board's liaisons to the committee, made important contributions to advancing and uplifting the debate across a range of critical issues. Yolanda Rodriguez and Betty Burton served as the lead staff for the committee, uncovering difficult issues and developing solutions when the committee reached an impasse. Cathy Seeley, as principal writer, employed her significant talent with words to develop the committee's distinct perspectives, and she contributed significantly to the development of content in her role as advisor. Staff member Jacqueline Olkin's careful eye for editing helped enhance the report's clarity, and David Merline oversaw the final stages of production. This effort also benefited from the good counsel of the National Council of Teachers of Mathematics and particularly its president, Gail Burrill.

In the end, the National Board for Professional Teaching Standards takes full responsibility for these standards, but they would not have received as positive a reception as they have without the wisdom, intelligence, and care of those who have willingly given their time and energy to this landmark effort to improve American education.



## The core propositions of the National Board for Professional Teaching Standards

- 1) *Teachers are committed to students and their learning.*
- 2) *Teachers know the subjects they teach and how to teach those subjects to students.*
- 3) *Teachers are responsible for managing and monitoring student learning.*
- 4) *Teachers think systematically about their practice and learn from experience.*
- 5) *Teachers are members of learning communities.*